Nonlinear Science Seminar: Effects of Singularities

Abstract

Philip J. Morrison (University of Texas at Austin)

Title: Hamiltonian Description of Matter I

Abstract: The Hamiltonian form and its intimately aligned action principle have served as an encompassing mathematical template for physical law. Examples include interacting particles, with the canonical Hamiltonian form and symplectic geometry, or the more exotic Hamiltonian form based on the noncanonical Poisson bracket, with Lie algebraic properties and Poisson geometry. Properties of the latter, the generic form for matter models, will be surveyed, both for finite-dimensional and infinite-dimensional (field) theories. The origin of noncanonical Poisson brackets via reduction and symmetry will be described.

Title: Hamiltonian Description of Matter II

Abstract: Several examples of noncanonical Hamiltonian theories will be presented. Applications to intriguing finite-dimensional systems based on finite-dimensional Lie algebras, such as the Kida vortex problem and the rattleback toy will be described, and infinite-dimensional systems such as magnetohydrodynamics (MHD), extended MHD, Vlasov kinetic theory, and other matter models will be discussed. The role of Hamiltonian theory in model building, stability theory, and computational algorithm development will also be discussed.

Tatashi Tokieda (Stanford University)

Chain reactions:

To every action corresponds an equal and opposite reaction. However, there turn out to exist in nature situations where the reaction seems neither equal in magnitude nor opposite in direction to the action.

We will see a series of table-top demos and experimental movies, apparently in more and more violation of Newton's 3rd law, and give an analysis of what is happening, discovering in the end that the phenomenon is in a sense generic. The keys are shock, singularity in the material property, and supply of `critical geometry'.

Vlad Vicol (Princeton University)

Mathematical aspects of hydrodynamic turbulence:

Motivated by theories of hydrodynamic turbulence, we consider dissipative weak solutions to the 3D incompressible Euler and Navier-Stokes equations. By combining ideas from convex integration with rescalings consistent with a multifractal intermittent turbulent flow, we construct solutions of the 3D Euler and Navier-Stokes equations that have anomalous dissipation of kinetic energy, and exhibit a highly nonunique character. We also discuss bounds for the second order structure functions which deviate from the classical Kolmogorov 1941 theory.

Richard Tsai (University of Texas at Austin)

The level set method and related algorithms for treating singularities that arise in the applications:

In the lectures, I will first review a few important components of a standard level set method, with special attention to treating potential singularities in the solutions. Since in general, one aims at computing the viscosity solutions of the derived formulation, it is essential to deal with singularities that arise naturally. Secondly, we shall discuss some recent development in computation-friendly formulations of boundary integral methods and variational principles on hypersurfaces via suitable extensions of the involved integro-differential operators. Finally, we shall also discuss a new class of algorithms dealing with crystalline curvature flows for spiral curves — a case which include singular terms in the partial differential equations as well as in the solutions.

Elena Tobisch (Johannes Kepler University Linz)

D-cascade: mathematical model and examples of the energy spectrum computation.

I present briefly the mathematical model of dynamical cascade (D-cascade) formed in wave systems with moderate nonlinearity due to modulation instability (which is another name for the Benjamin-Feir instability). I demonstrate how to compute the shape the D-cascade energy spectrum taking as examples various types of water waves. Moderate nonlinearity means e.g. for surface water waves that wave steepness is of the order of 0.1 (cf.: in the wave turbulence theory it is of the order of 0.01).

D-cascade: experimental observations and open questions.

I present results of the laboratory experiments with water waves where the D-cascade was clearly observed, for a wide range of the excitation parameters. I also formulate a few open problems - coming both form the theory and experiment - whose solution will bring us a deeper understanding of the behavior of non-linear wave systems with moderate nonlinearitiy.