# Remarks on pronounced non-linear characteristics of transport in fusion plasmas

## - from experimentalist's view point -

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*"While nature creates a curve,*
*the human creates a straight line"*

Communications with and materials by Prof.K.Itoh, Dr.M.Kikuchi and Prof.Z.Yoshida are gratefully acknowledged.

# Outline

1 Introduction

“Fusion plasma is non-equilibrium open system”

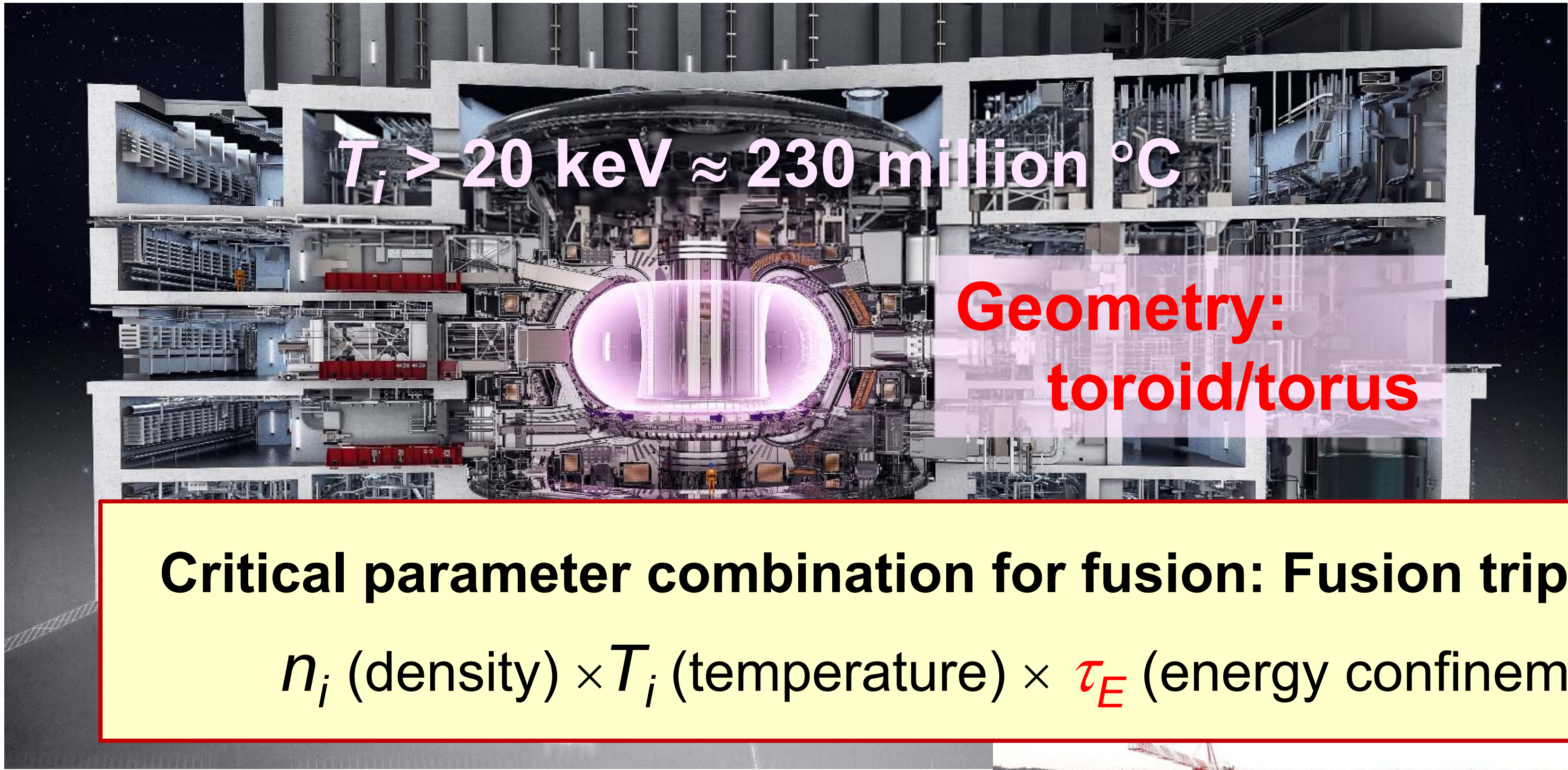
2 Global energy confinement time and heat transport

“Diffusion equation”

3 Defect of diffusion equation against experimental observations and modification / improvement

- Critical gradient
- Loss of spatial symmetry
- Bifurcation
- Non-locality

4 Summary



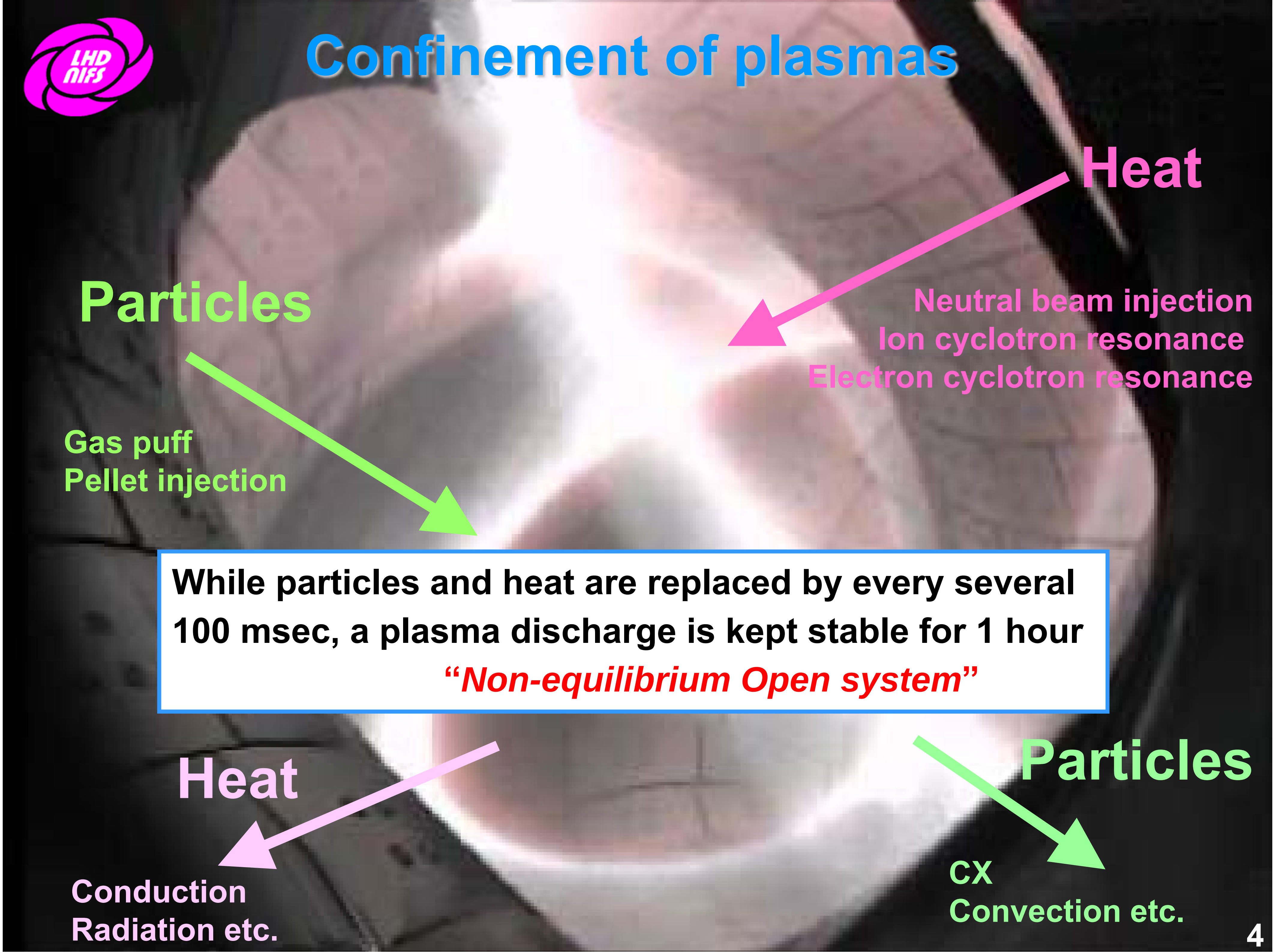

**Critical parameter combination for fusion: Fusion triple product**

$n_i$ (density) $\times T_i$ (temperature) $\times$ $\tau_E$ (energy confinement time)

$$n_i T_i \, \tau_E \approx 4 \times 10^{21} \text{ m}^{-3} \text{ keV s}$$

***"Gimme a number"*** *mentality*

⇔ *Concern to lose chance to comprehend this fascinating object (fusion plasma) and to enhance scientific predicting capability*



# Confinement of plasmas

**Particles**

Gas puff
Pellet injection

**Heat**

Neutral beam injection
Ion cyclotron resonance
Electron cyclotron resonance

**While particles and heat are replaced by every several 100 msec, a plasma discharge is kept stable for 1 hour**

***"Non-equilibrium Open system"***

**Heat**

Conduction
Radiation etc.

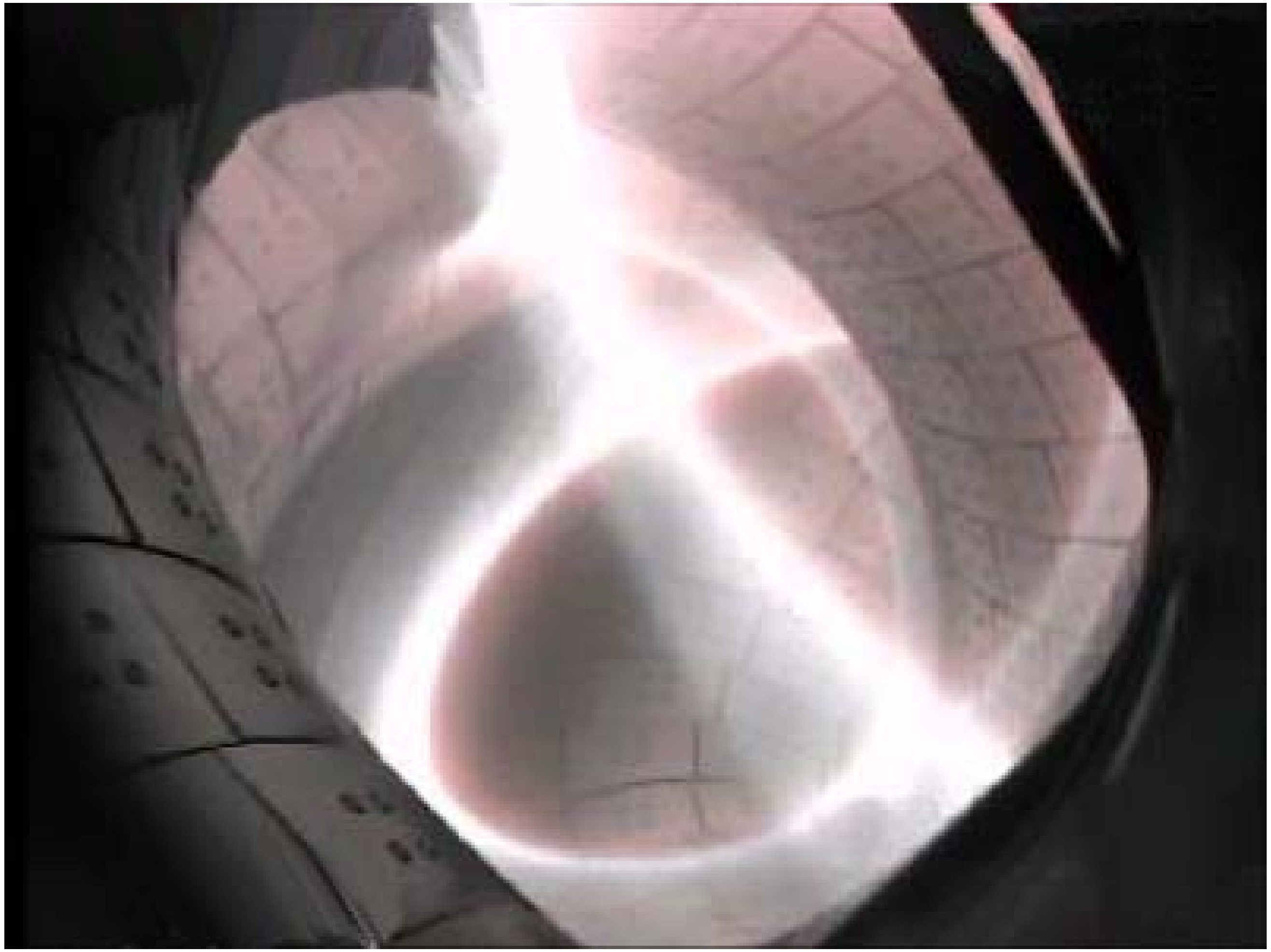
**Particles**

CX
Convection etc.

# Plasma in Large Helical Device at NIFS (Toki, Japan)

Looks like steady-state and stable,
but this is typical non-equilibrium open system ➔ highly non-linear

- **Closed system** ➔ thermo-dynamic equilibrium (trivial state)
- **Open system** ➔ can stand apart far from thermo-dynamical equilibrium by external driving forces



*I.Prigogine*
"Dissipative structure"

**Today's my talk**

**Transport: diffusive nature**

# Concept of energy confinement time

- Store water in a bucket with holes

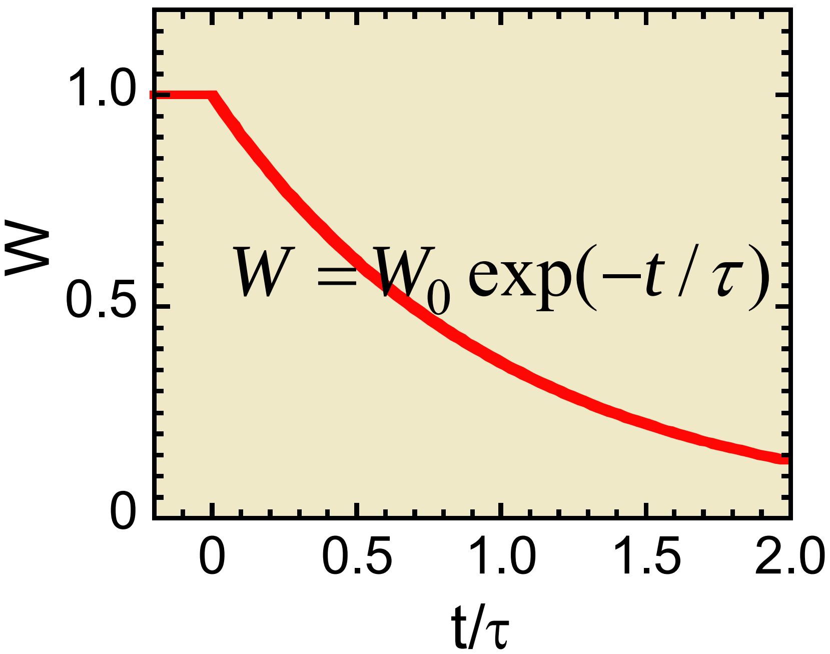
| | |
|---|---|
| Inlet flow | P $m^3/s$ |
| Loss flow | L $m^3/s$ |
| Stored water | W $m^3$ |

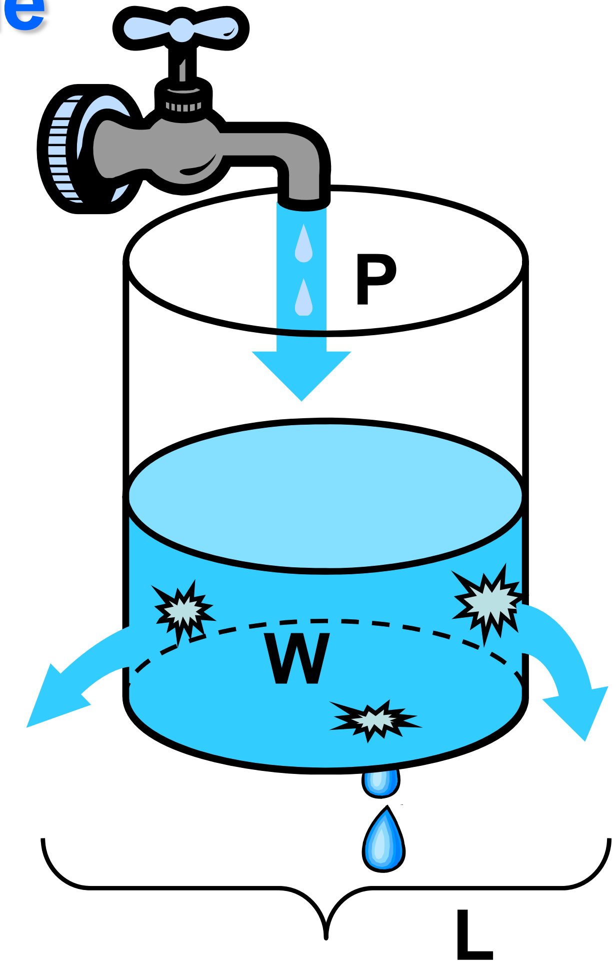
- Temporal change of W : $$\frac{\mathrm{d}W}{\mathrm{d}t} = P - L$$

L is proportional to W since the pressure to drive L is proportional to W ➔

$$L = cW = W/\tau$$

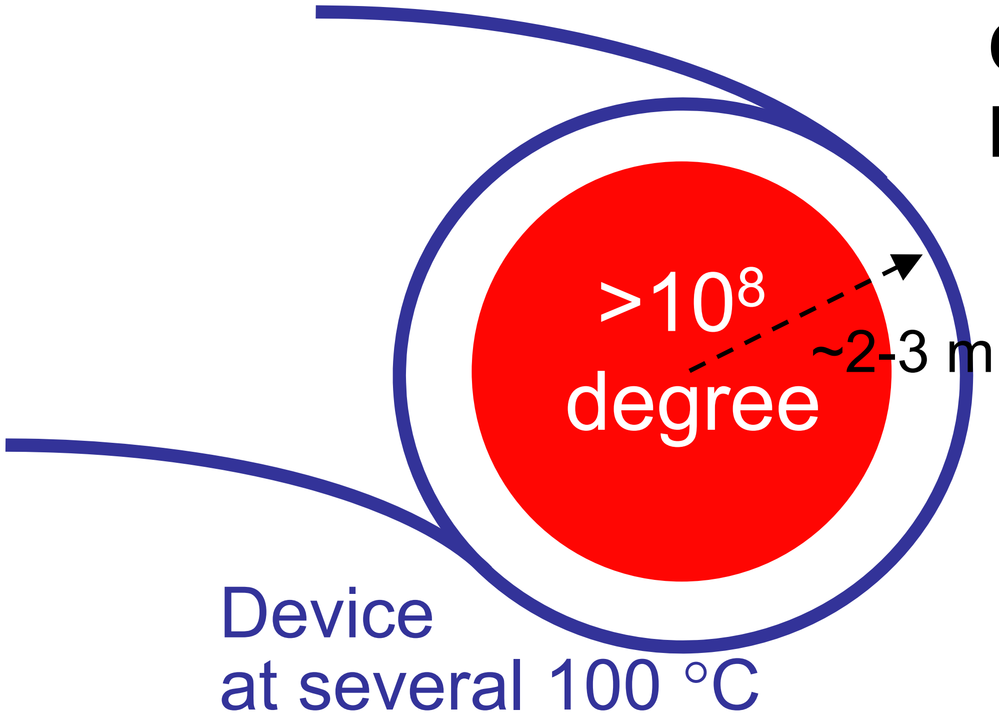
$$\frac{\mathrm{d}W}{\mathrm{d}t} = P - \frac{W}{\tau}$$




In plasma confinement
*W* : stored energy
*P* : heating power
$\tau_E$ : global goodness of confinement
➔ energy confinement time
in steady state $\tau_E = W/P$

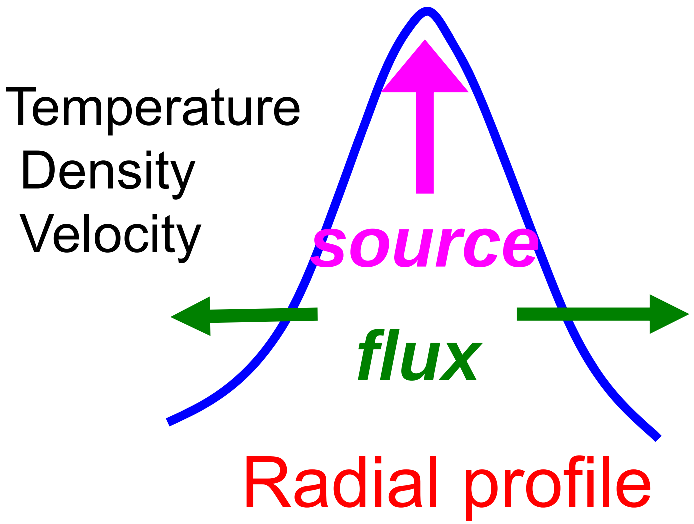
# Plasma Confinement




**Challenging requirement:**
**Extreme thermal insulation**

1 M ºC/cm

Steep gradient causes turbulence
Temperature $\nabla T$, Density $\nabla n$




**Radial profile : Structure**

← formed and maintained by characteristics of non-linear, non-equilibrium system

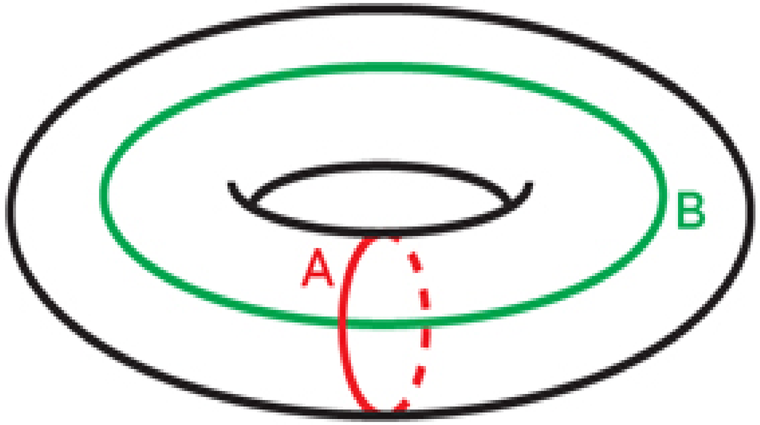
# Relation between global confinement (0-D) and heat transport (1-D)

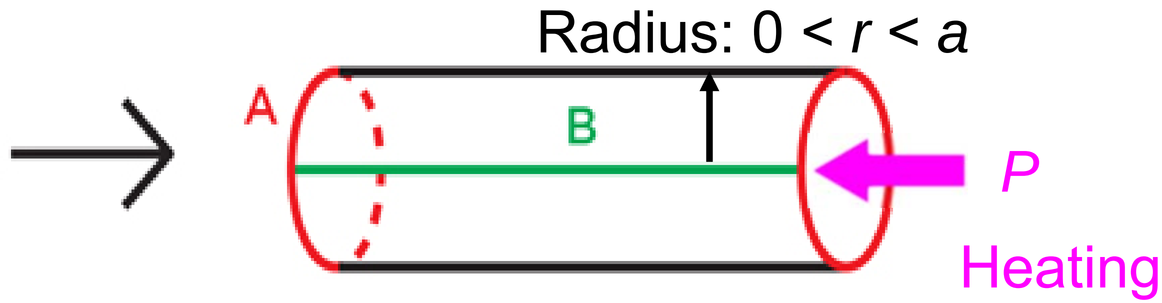
**Fick's law** $q = \kappa \nabla T$

Flux = Coefficient × Local gradient

**Diffusion equation of heat**

$$\frac{3}{2} n \frac{\partial T}{\partial t} = \nabla \cdot (n \chi \nabla T) + P$$




Separation of variables $T = T(r)\exp(-t/\tau)$

$$\frac{\mathrm{d}^2 T}{\mathrm{d} r^2} + \frac{1}{r}\frac{\mathrm{d} T}{\mathrm{d} r} + \frac{3}{2}\frac{1}{\chi\tau} T = 0$$

Bessel's differential equation

$$T = \mathrm{J}_0(r/(a/2.4))\exp(-t/\tau) \qquad \frac{2}{3}\chi\tau = \frac{a^2}{2.406^2}$$

$$\Rightarrow \chi\tau = a^2/3.9$$

Local heat diffusivity × Decay (Confinement) time = Size$^2$

# Discovery and evolution of transport picture as experiments approach to fusion condition

In 1970', simple diffusive picture was acceptable

(because *P* was small enough, namely $\Delta T$(or $\nabla T$) was small)

In 1980', this picture showed poor prediction for larger *P* ($\Delta T$ (or $\nabla T$) )

$\tau_E \propto P^{-1/2}$ ➔ $\chi \propto T^{3/2}$ so called "power degradation"

Diffusion equation becomes non-linear (but still local) like

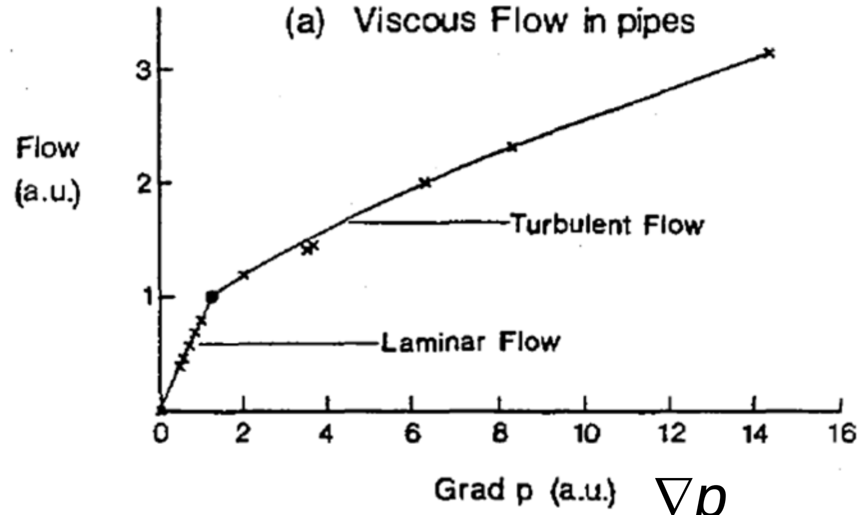
$$q/n \propto T^{\alpha} \left|\nabla T\right|^{\beta}$$

Since then, peculiar phenomena, which sometimes led to breakthrough and sometimes led to further difficulty, have been discovered

- Critical gradient
- Loss of spatial symmetry
- Bifurcation
- Non-locality

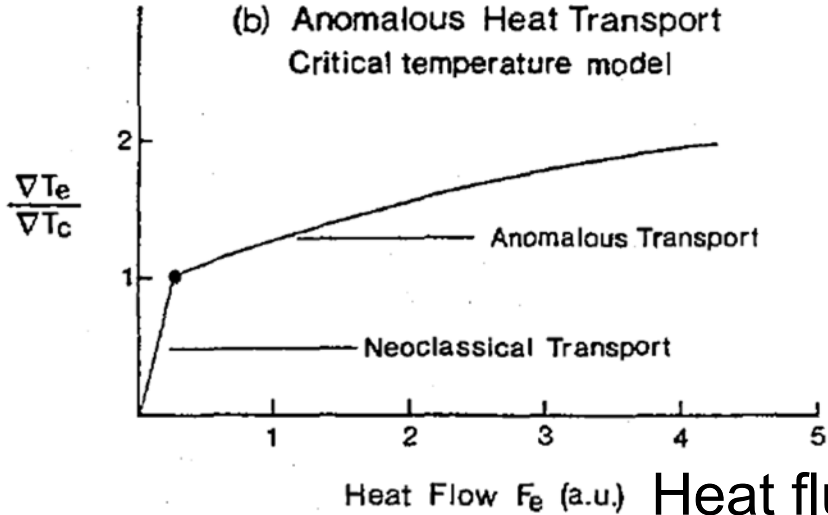
# Non-linear behavior of heat diffusivity $\chi$

Flow




$\nabla p$

$\nabla T / \nabla T_c$




Heat flux

## Viscous flow in pipes

When Reynolds number, R, reaches the critical value, extra resistance is added to the flow which increases with the value of R.

## Heat transport in fusion plasmas

The dependence of heat flow on the temperature gradient shows the same behavior. When $\nabla T$ reaches $\nabla T_c$, additive transport appears which increases the heat flux. This additive transport also varies non-linearly with the ratio $\nabla T / \nabla T_c$.

Heat flux

H: Heaviside step function

$$F_e = n_e \chi_{an,e} \nabla kT_e \left[1 - (\nabla kT_e)_c / (\nabla kT_e)\right] H\left[\,|\nabla kT_e| - |\nabla kT_e|_c\,\right] H\left[\nabla q\right] + F_{e,neo}$$

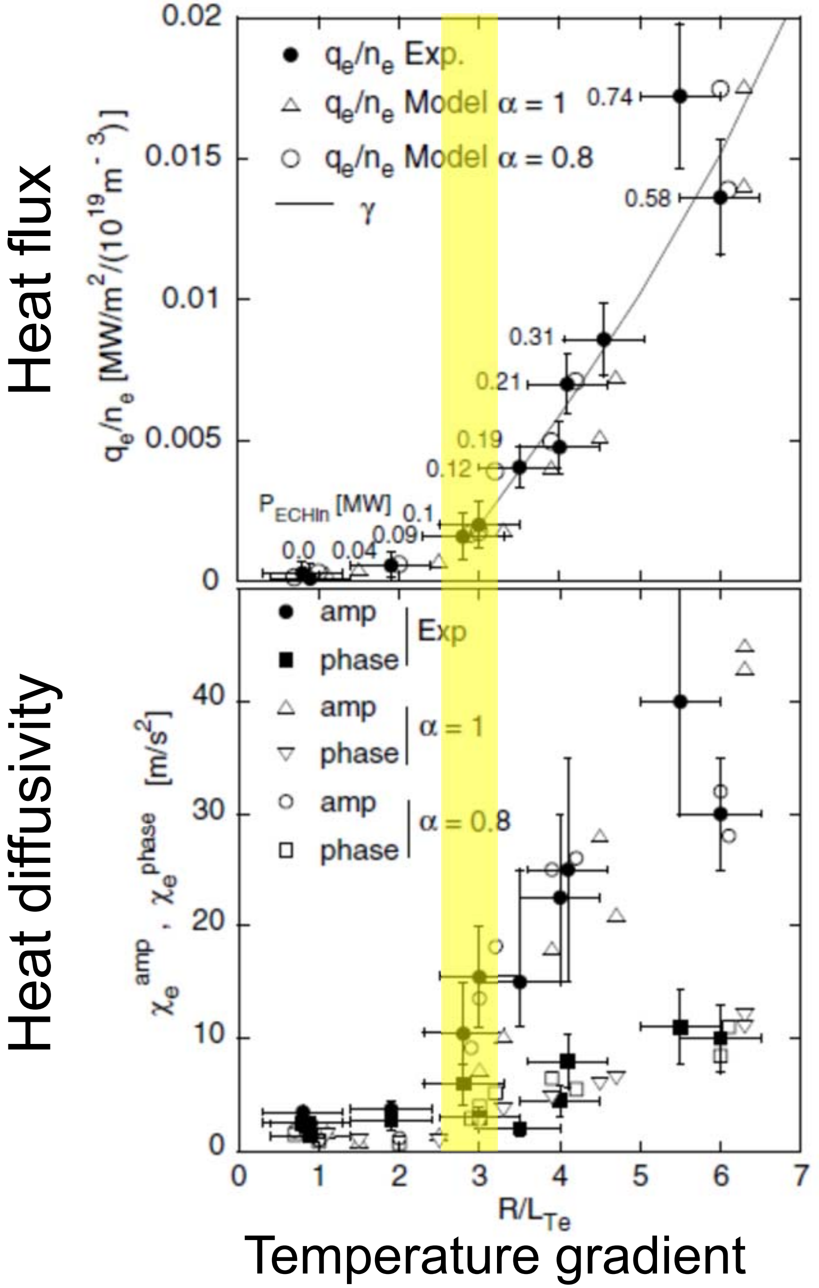
$$\chi_{an,e} = 0.15 \left[(\nabla kT_e / kT_e) + (2\nabla n_e / n_e)\right] (T_e/T_i)^{1/2} (R/r) (q^2/\nabla q B R^{1/2})\, c^2 (\mu_0 m_i)^{1/2}$$

$$(\nabla kT_e)_c = 0.06\left[\eta J B^3 / n_e (kT_e)^{1/2}\right]^{1/2} (1/q) \left(e^2 / \mu_0 m_e^{1/2}\right)^{1/2}$$

P.Rebut, P.P.Lallia, M.L.Watkins, Plasma Phys. Control. Fusion Res. 1988, IAEA, Vol.2, p.191

# Experimental validation of existence of critical temperature gradient

AUG (DE)

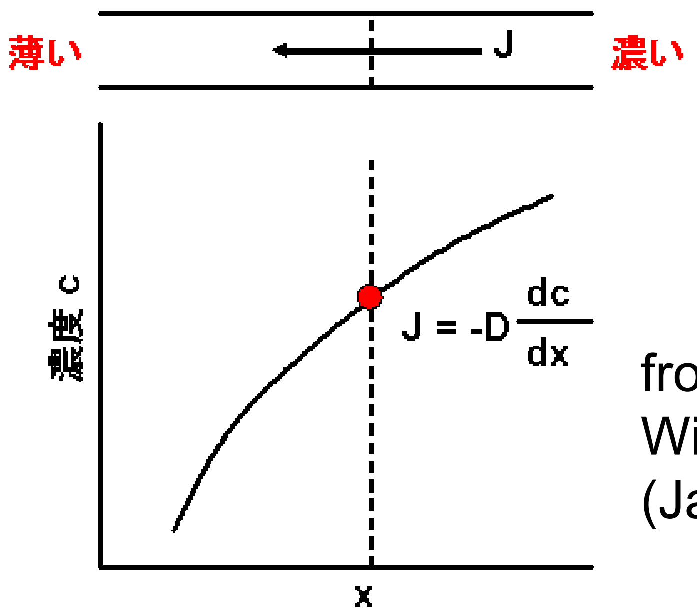



R.Ryter et al., Phys. Rev. Lett. (2005)

Non-linearity is not expressed by simple power law like

$$q/n \propto T^{\alpha} \left|\nabla T\right|^{\beta}$$

Still local picture: Local quantities, such as temperature, temperature gradient are deterministic in dynamics




from “Fick’s law” Wikipedia (Japanese)

# Transport matrix for plasma

**Curie's principle** : linear relation between flow and thermo-dynamical force

Also linear coupling between different flow and force is seen in nature
ex. Seebeck effect

Diffusion equation is a part of transport matrix
Diffusive nature is pronounced when diagonal term is dominant

Flux — Diffusion coefficient — Thermo-dynamical force : gradient

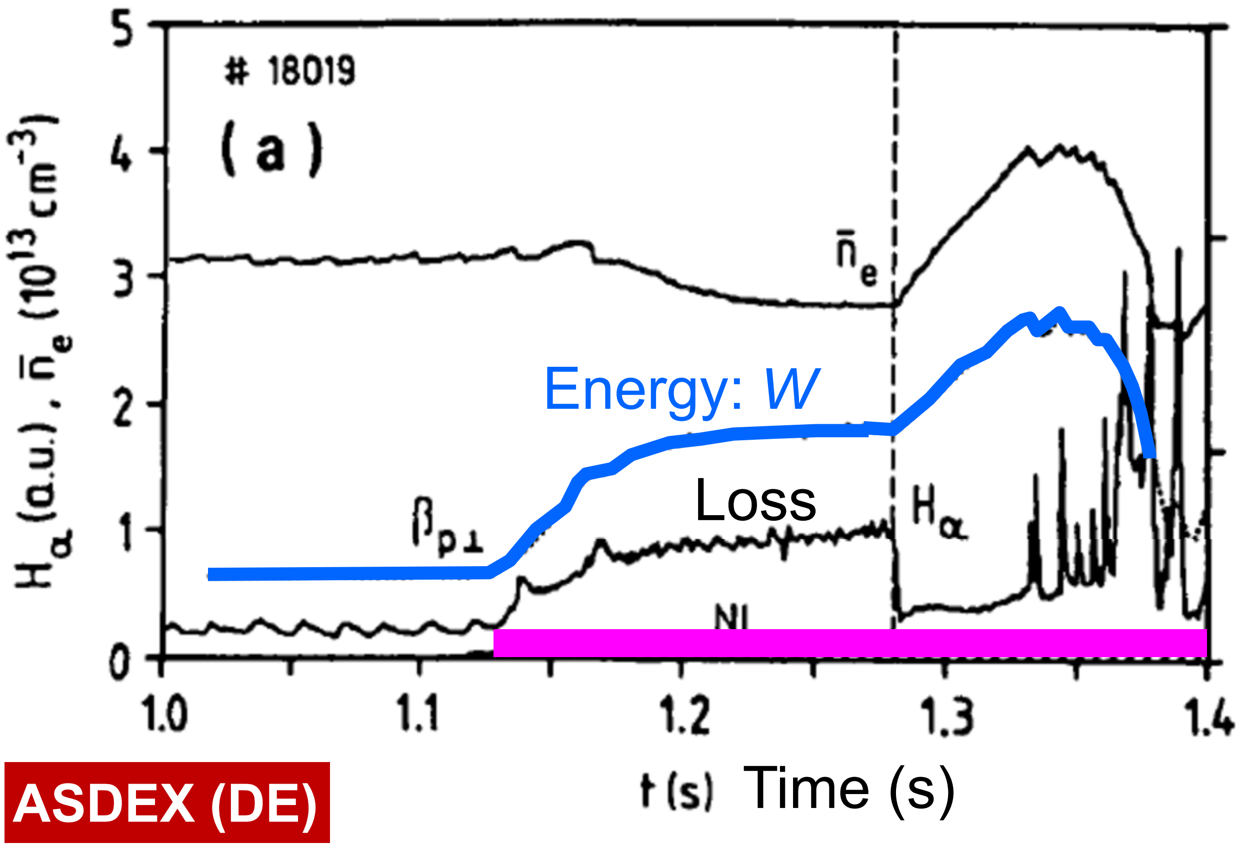
$$\begin{pmatrix}\Gamma \\ P_\phi \\ q_{i,} \\ q_{e,}\end{pmatrix} = - \begin{pmatrix} D & - & - & - \\ - & \mu_\phi n m_i & - & - \\ - & - & n\chi_{i,} & - \\ - & - & - & n\chi_e \end{pmatrix} \begin{pmatrix}\nabla \underline{n_e} \\ \nabla V_\phi \\ \nabla T_i \\ \nabla T_e\end{pmatrix}$$

| | Flux | Gradient | |
|---|---|---|---|
| particle | $\Gamma$ | $\nabla \underline{n_e}$ | → Non-Diffusive |
| toroidal momentum | $P_\phi$ | $\nabla V_\phi$ | → Non-Diffusive |
| ion heat | $q_{i,}$ | $\nabla T_i$ | → Diffusive |
| electron heat | $q_{e,}$ | $\nabla T_e$ | → Diffusive |

In plasmas, off-diagonal terms often play essential roles :
← difficulty in finding **orthogonal** basis

Loss of spatial symmetry → formation of non-trivial structure

# Bifurcation:   from "gradient curve" to "flux landscape"




**ASDEX (DE)**

Energy evolves in line with

$$W = P\tau + (W_0 - P\tau)\exp(-t/\tau)$$

And $\chi$ degrades with *T* due to *P*

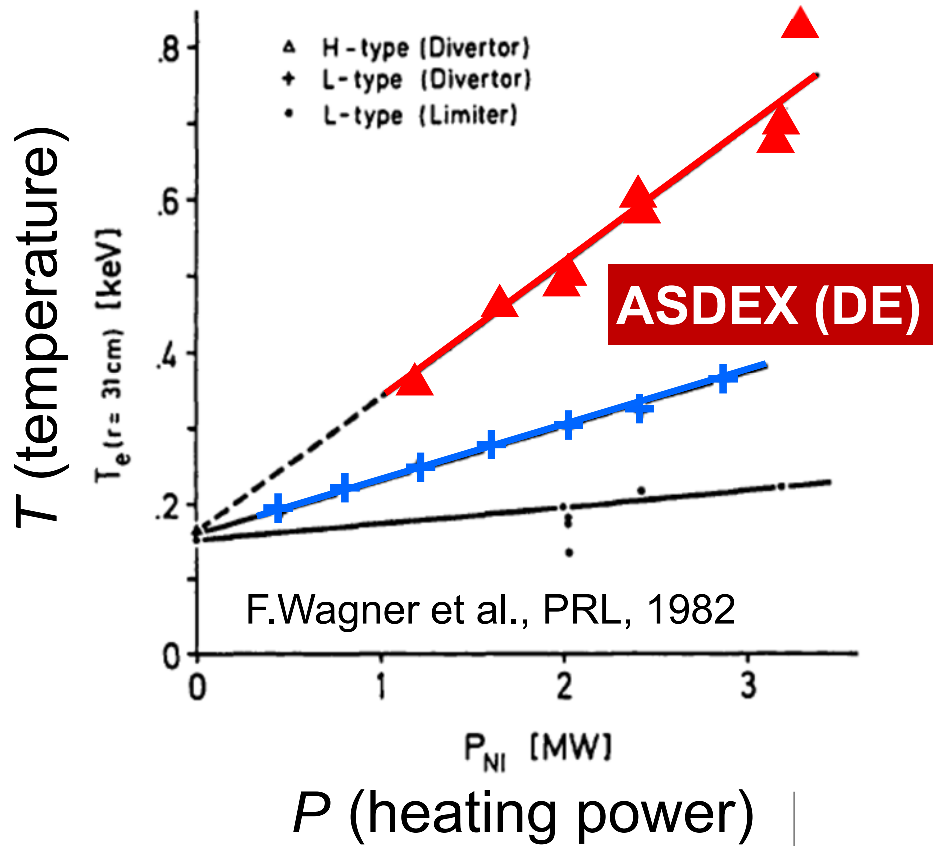
Then, sudden change happens

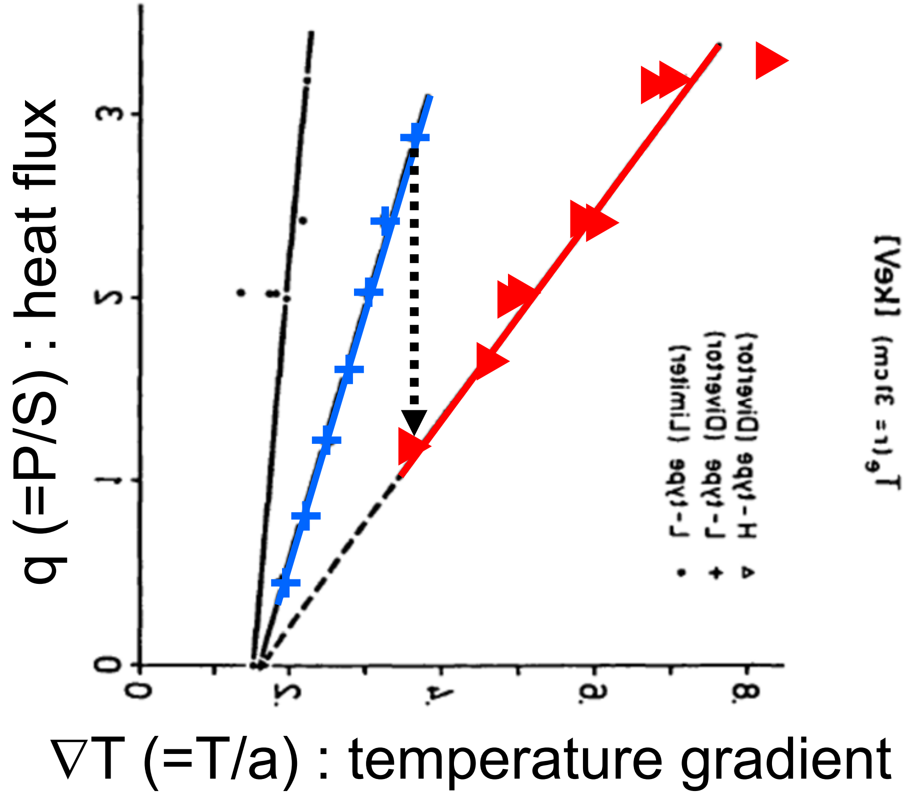
Heating power: *P*

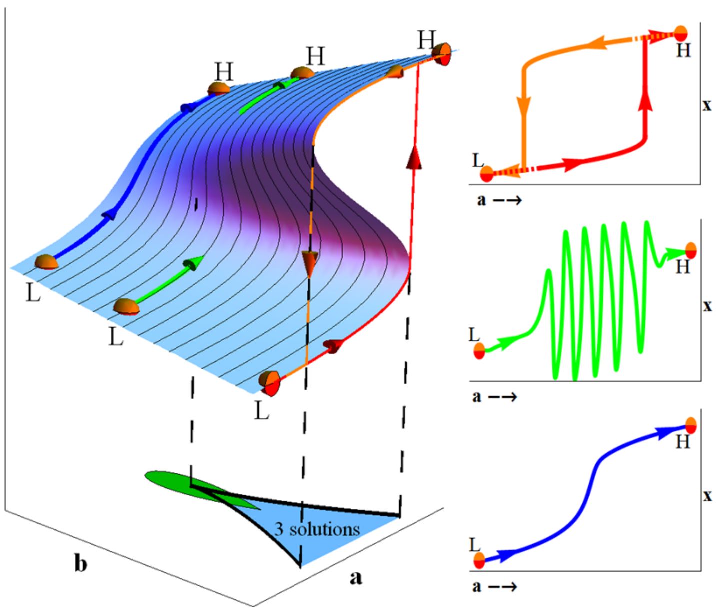
F.Wagner et al., "*Regime of **improved confinement** and high beta in neutral-beam-heated divertor discharges of the ASDEX tokamak*", PRL, 1982    # of citation = 1311 by Scopus

J.B.Taylor "*Relaxation of toroidal plasma and generation of reverse magnetic field*" PRL, 1974  # of citation = 1042 by Scopus

# Bifurcation: from "gradient curve" to "flux landscape"

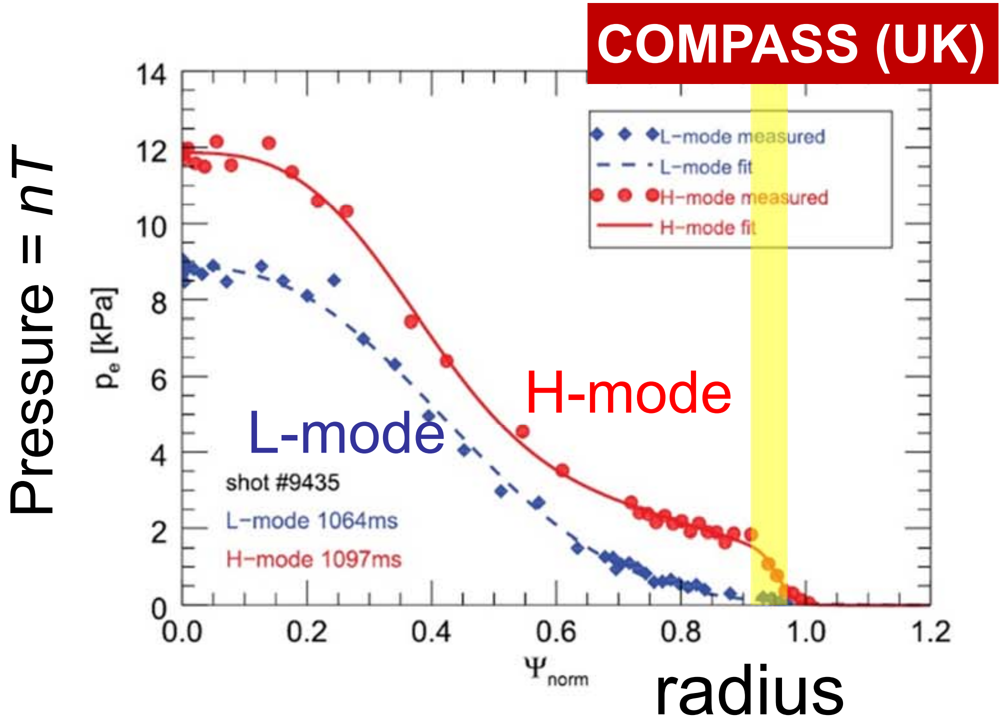


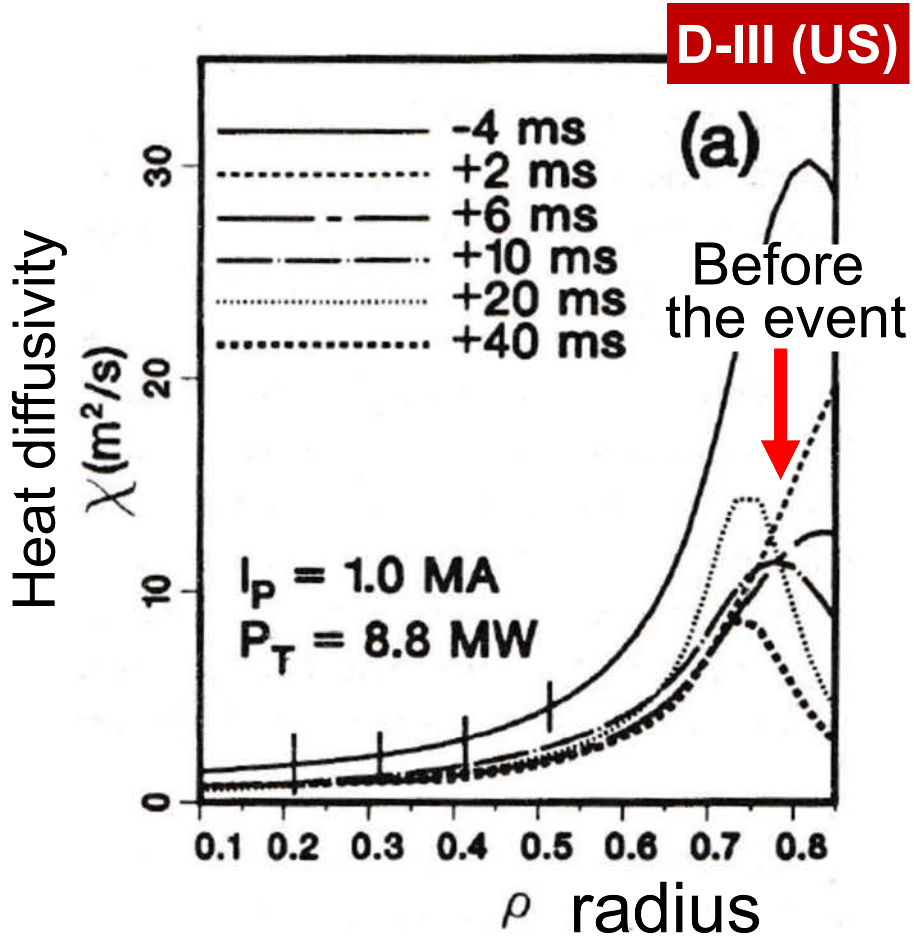




Heat transport exhibits hysteresis

# This sudden spontaneous change is very peculiar from usual diffusive nature




P.Pánek et al., Plasma Phys. Cont. Fusion, 2015




K.H.Burrell et al., Plasma Phys. Control. Fusion Res. 1990, IAEA, Vol.1, p.123

Space: very narrow ➔ barrier
Time: much faster than diffusion time ➔ transition
Trajectory on parameter space ➔ threshold power, hysteresis

# Non-local-in-space model for fast response

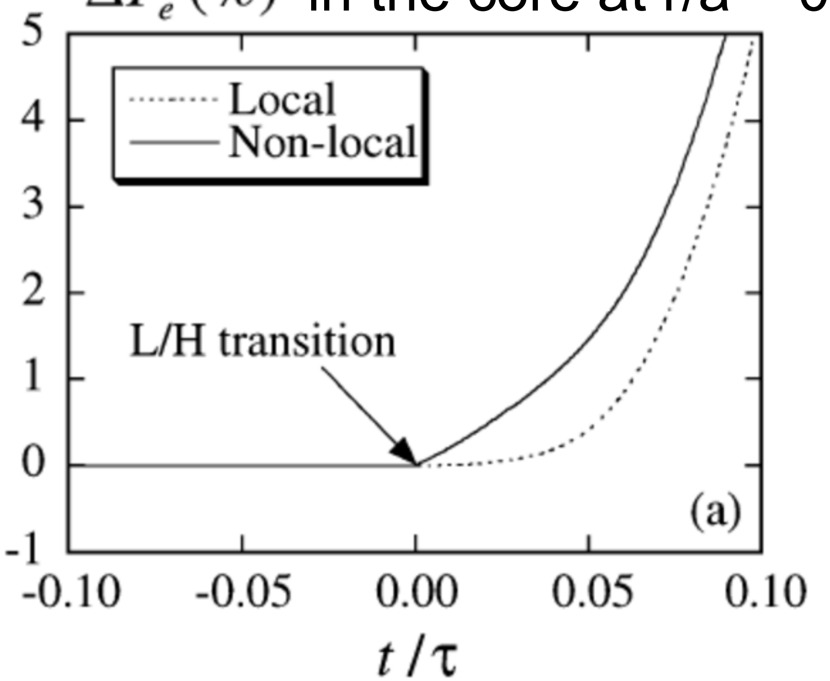
$$q(r,t) = -\int_0^r n(r',t)\chi(r',t)K(r',r)\nabla T(r',t)dr'$$

*K* (*r'*,*r*) is the kernel to produce non-local effect

Physical quantities like *T* and ∇*T* at *r'* could be influential if distance |*r*-*r*'| is shorter than radial correlation length ℓ.

For example, $K_\ell(r,r') \equiv \frac{r}{r'}\left[C_{local}\delta(r-r') + C_{non-local}\frac{1}{\sqrt{\pi}}\exp\left\{-\left(\frac{r-r'}{\ell}\right)^2\right\}\right]$

$\Delta T_e$ (%) in the core at r/a = 0.1



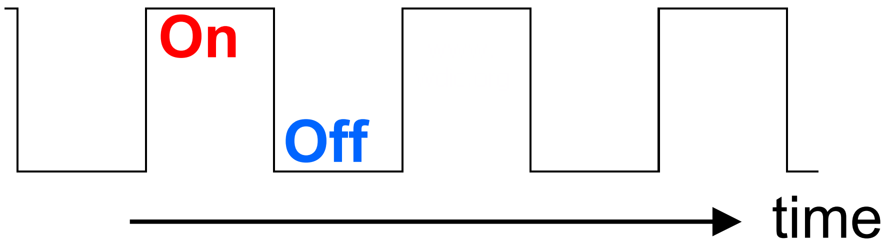

χ at *r* = *a* (edge) is forced to change at *t*=0

Core temperature responds faster than in diffusion process

T.Iwasaki et al., J. Phys. Soc. Jpn (2000)

# Multiple state in heat flux vs local gradient

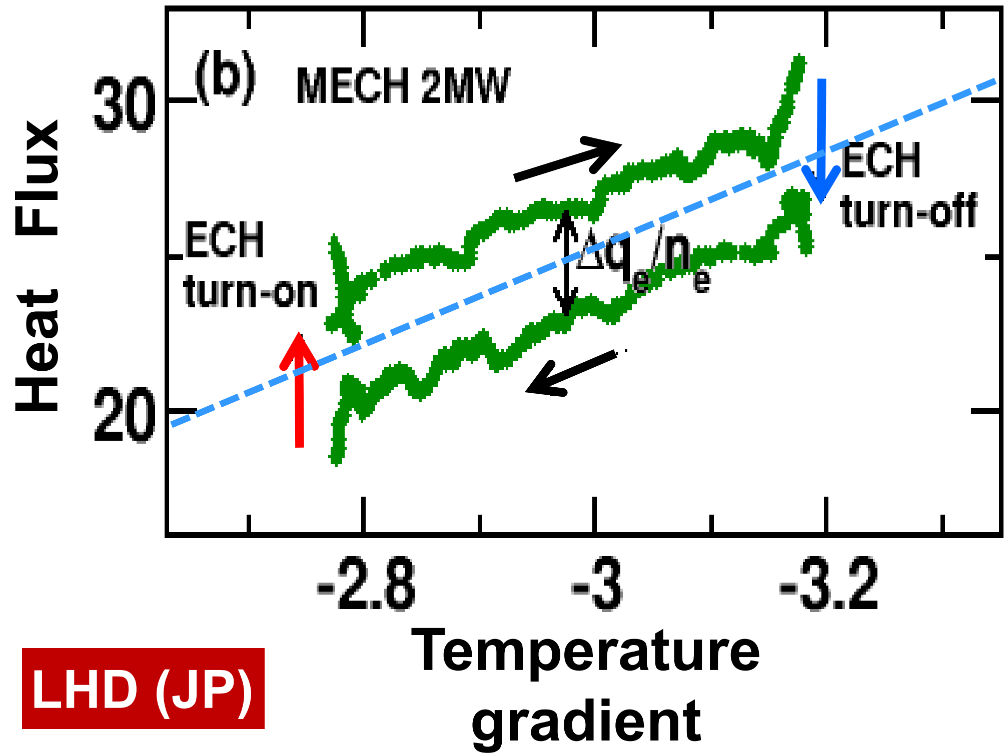
Additional heating power




Two time-scales:

abrupt change when heating turned on/off

slow change with global parameters (temperature profile)




LHD (JP)

**Hysteresis in flux-gradient relation**

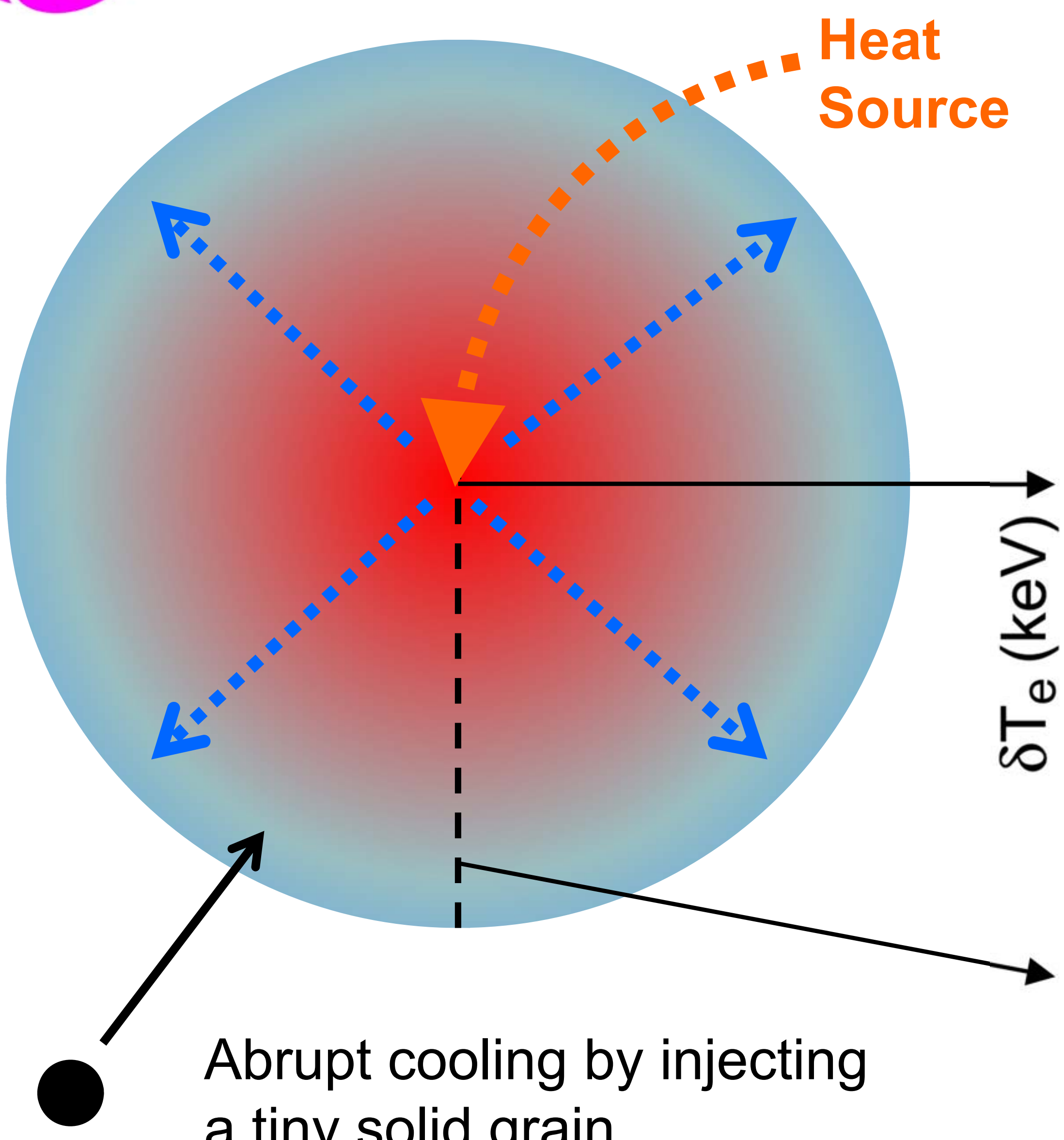
$$\delta q_e = -\frac{1}{S}\int\left(\frac{3}{2}n_e\frac{\partial \delta T_e}{\partial t} - \delta p_{ECH}\right)dV$$

*Heat flux can change without change of local temperature and temperature gradient*

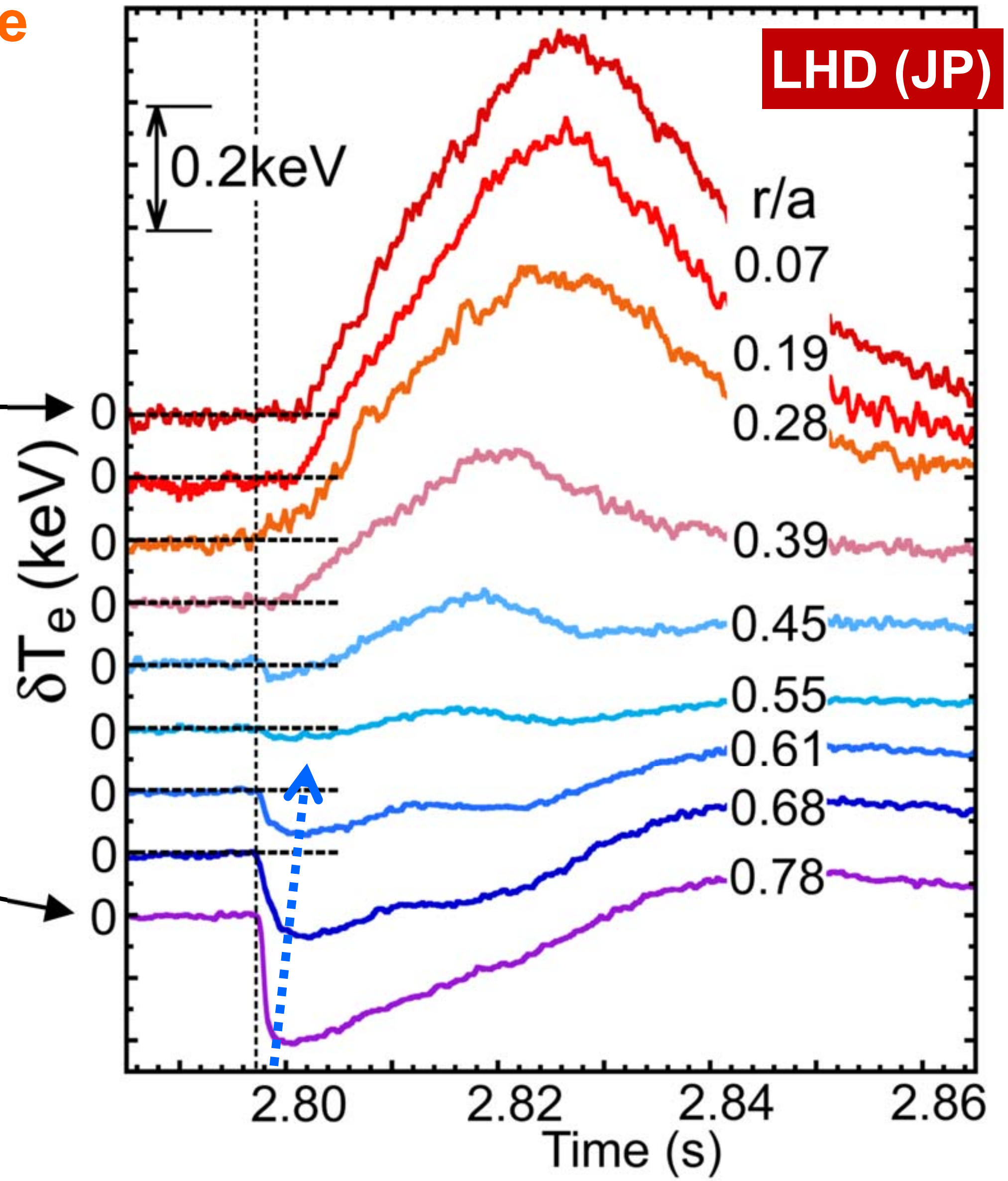
**➔ Local (classical) theory (Heat) flux ∝ (Temperature) gradient is violated**

S.Inagaki et al., Nucl. Fusion, 2013

# Non-local and non-linear response of plasma




N.Tamura et al., Nucl. Fusion, 2007




Further enhanced predicting capability is requisite to control fusion plasma

# 3N's : Non-linear, Non-orthogonal, Non-local

*simplified* *unrealistic ?*

**1 Linear + diffusive + local transport model (simple!)**

$q/n = -\chi \nabla T$ $\Gamma = -D \nabla n$

**2 Add Non-linearity**

$q/n = -\chi(T, \nabla T)\, \nabla T$ $\Gamma = -D(n, \nabla n)\, \nabla n$

Non-liner dependence of diffusivity → critical gradient, stiffness

**3 Add Non-diffusivity (inter-linkage between different gradients)**

$q/n = -\chi(T, \nabla T)\, \nabla T - D^{N}(n, \nabla n) \nabla n + \cdots$

Non-diffusive term → inward/outward pinch

**4 Add Non-locality (inter-linkage in space)**

$q/n = -\chi(T, \nabla T)\, \nabla T - D^{N}(n, \nabla n) \nabla n + \int \chi\, K(r') \nabla T\, (r') dr'$

Non-local term

**5 Incorporation of new thermo-dynamical force ?**

*complicated ?* *realistic ?*

## Déconstruction
## or Copernican revolution ?

Plasmas are gentle in steady-state or slow change

➔ Diffusive model is successful to significant extent which should be commended.

However, we should be humble and prepare risks

Gimme a number mentality tends to incline to easy way.

Pay more attention to what a model agrees to

Pay less attention to what a model does not agree to

In particular, how about transient / short-lifetime phase?

operation to lead to ignition

instabilities

external perturbation

➔ Feedback control of non-linear system for safe operation of burning plasma

# Summary

- Success story of diffusive picture seems to be declining as fusion plasma performance is improved.
- Diffusive picture is not sure to fulfill “Gimme a number” mentality (for good prediction and control) even in burning plasmas.

**from Ordnung der Wirklichkeit**
**(Reality and Its Order,「真理の秩序」)　by W.Heisenberg**

“An idea ・・・・・ is not meant to be as faithful a representation of reality as possible（忠実な模写）but to be the seed for further series of ideas（思考の鎖の要）. The issue is not the accuracy（正確さ）but the fruitfulness (豊かさ）of concepts. ・・・・・・Here, a sentence (“mathematics” HY) can, generally speaking, not be “right” or “wrong”. But one may call a sentence “true” when it fruitfully leads to an abundance of other ideas. The opposite of a “right” sentence is a “false” one. But the opposite of a ”true“ sentence will often be another “true” one.”