Hamiltonian Dynamics: Integrability, Chaos, and Noncanonical Structure

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Overview of Hamiltonian dynamics and its applications.

Basic Hamiltonian Dynamics

Why study Hamiltonian Dynamics?

- "Hamiltonian systems .... are the basis of physics."
  – M. Gutzwiller

- The most important equations of physics are Hamiltonian
  basic vs. applied

- Convenience and universality
  one function defines system and all have common properties

- Hamiltonian vs. Lagrangian
  Hamiltonian emphasized instead of action principle

Hamiltonian Systems

W. R. Hamilton for light rays 1824, for particles 1832

Hamilton's Equations:

$$\dot{q}^i = \frac{\partial H}{\partial p_i} = \{q^i, H\} \quad \text{and} \quad \dot{p}_i = -\frac{\partial H}{\partial q^i} = \{p_i, H\}\,, \qquad i = 1, 2, \ldots N\,,$$

Definitions:

$\dot{} = d/dt$
$H(q,p) =$ the Hamiltonian function
$q = (q^1, q^2, \ldots) =$ canonical coordinates
$p = (p_1, p_2, \ldots) =$ canonical momenta
$N =$ number of degrees of freedom (dimension/2)
N infinite $\Rightarrow$ Hamiltonian field theory

Poisson Bracket:

$$\{f, g\} = \frac{\partial f}{\partial q^i}\frac{\partial g}{\partial p_i} - \frac{\partial g}{\partial q^i}\frac{\partial f}{\partial p_i}$$

repeated sum notation

Hamiltonian Examples

“Natural” Hamiltonian Systems

$$H = \frac{p^2}{2} + V(q)$$

“Natural” Hamiltonian Systems

$$H = \frac{p^2}{2} + V(q) = \frac{1}{2} p_i \, g^{ij} \, p_j + V(q^1, q^2, \ldots, q^N)$$

g = metric tensor

- single particle in a potential, V
- problem of n interacting bodies
- planetary dynamics
- geodesic flow etc.

Natural Hamiltonian systems historically the most studied.

Advection in 2D Fluid Flow

neutrally buoyant particle or dye in given solenoidal velocity field

$$\boldsymbol{v}(x, y, t) = \hat{z} \times \nabla\psi(x, y, t)$$

moves with the fluid

$$\dot{x} = v_x = -\frac{\partial\psi}{\partial y} \quad \text{and} \quad \dot{y} = v_y = \frac{\partial\psi}{\partial x}$$

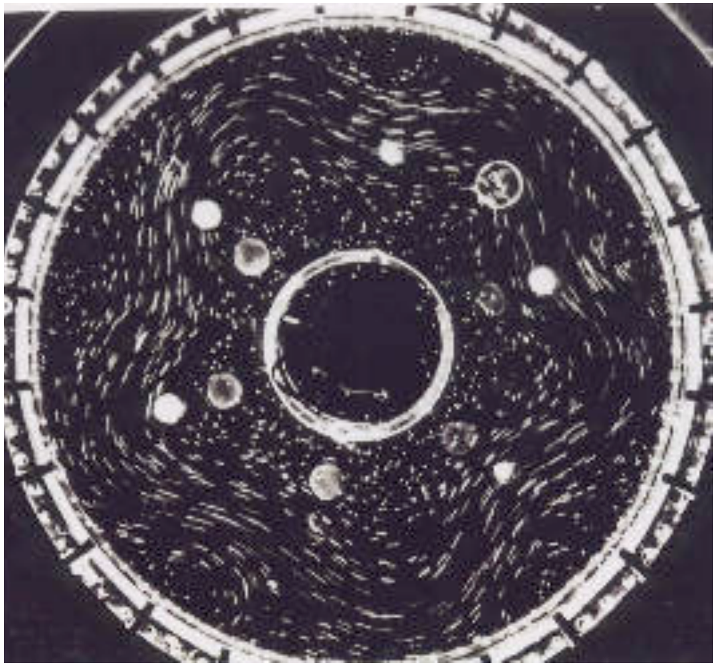
The Hamiltonian ψ need not have the "natural" separable form, but comes from solving a fluid dynamics problem and momentum is physically a coordinate!

Note: may be nonautonomous $\psi(x, y, t)$. If ψ is periodic in time it is common to say it counts as 1/2 degree of freedom. For example, here 1.5 DOF.

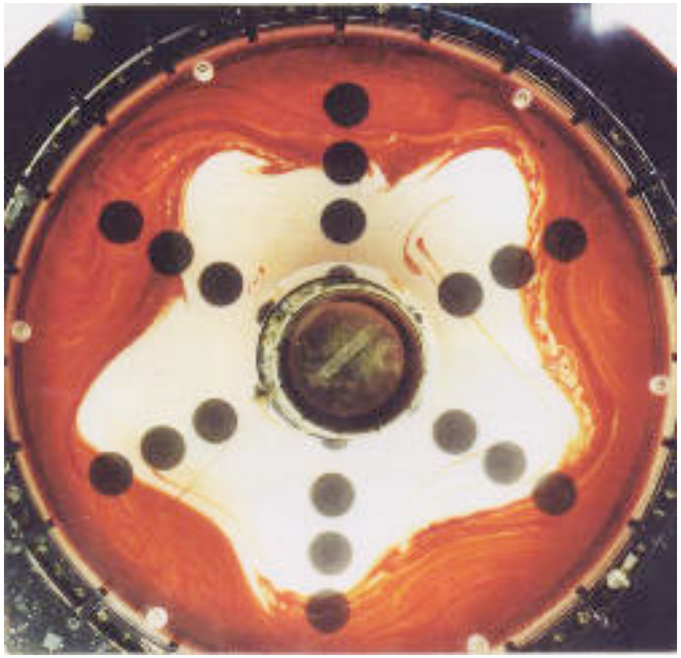
Swinney's Rotating Tank (circa 1987)

Cyclonic (eastward) jet

particle streaks

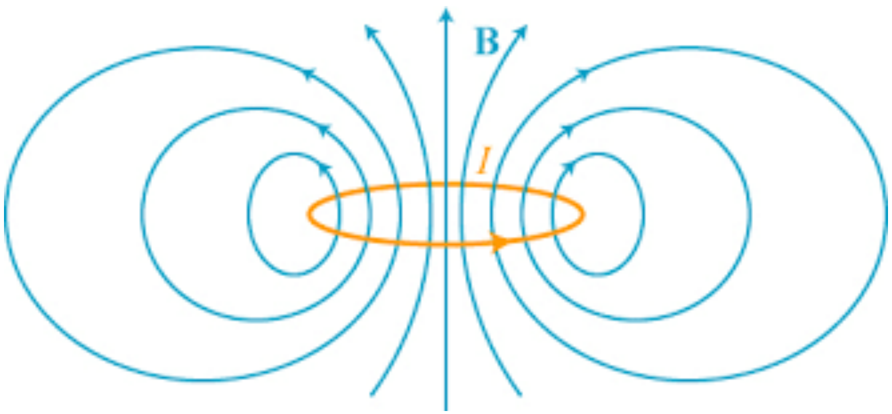


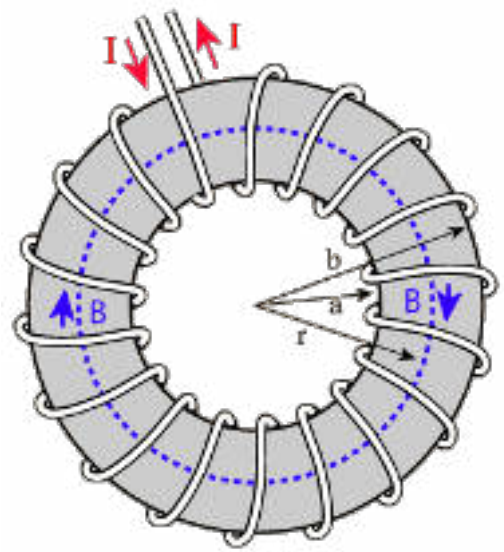
dye



Magnetic Field lines are Hamiltonian System

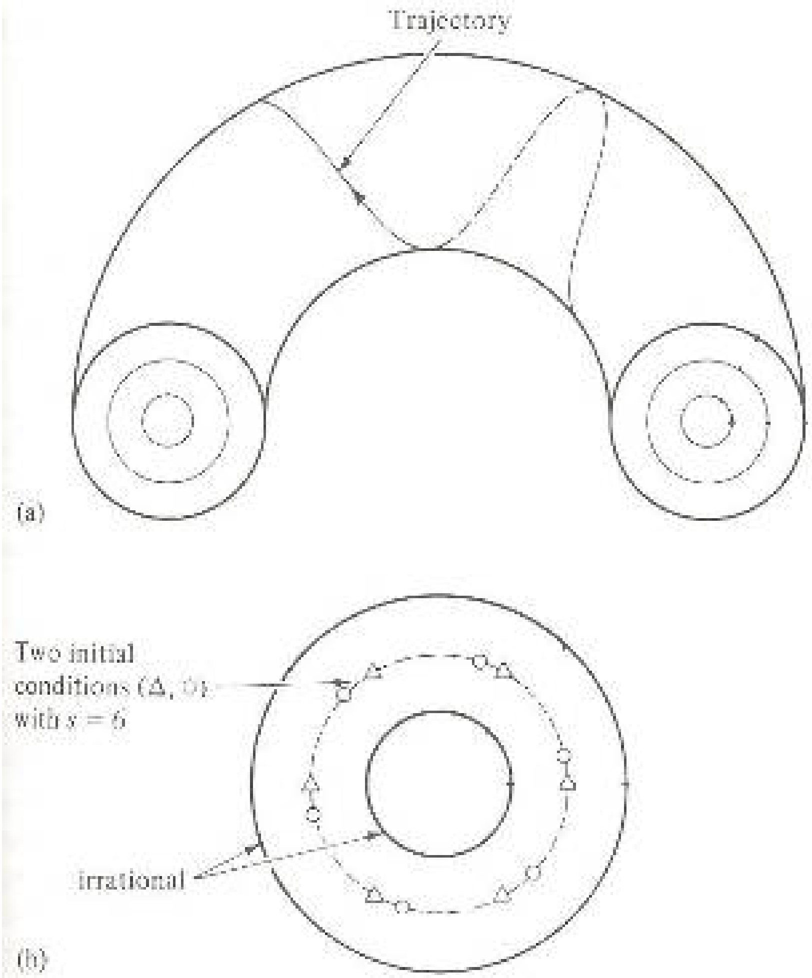
Examples: dipole and toroid





Time is coordinate.

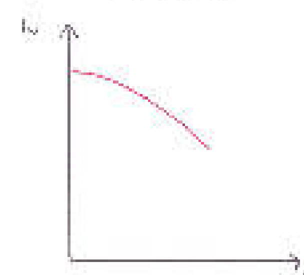
B-lines in Plasma Fusion Devices (cf. Dewar talk)

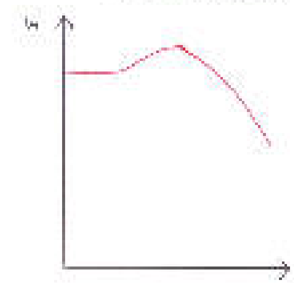


Two types of magnetic field lines:

- closed lines: periodic orbits (rational winding number: $\omega = t/s$)
- lines that cover the surface of a two-dimensional torus: quasiperiodic orbits (irrational winding number ω)

Examples of winding number profiles:





Cylinder or Straight Torus

Uniform guide field:

$$\boldsymbol{B} = B_0\,\widehat{z} + \widehat{z} \times \nabla\psi(x, y, z)$$

Magnetic field line equations:

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$

Use z as time-like coordinate, by setting $t = z/B_0$, then

$$\dot{x} = -\frac{\partial\psi}{\partial y} \quad \text{and} \quad \dot{y} = \frac{\partial\psi}{\partial x}$$

Works for both $z \in \mathbb{R}$ and $z \in \mathbb{T}^1$

Phase Space Coordinates

Dynamics takes place in phase space, $\mathcal{Z}$, which has coordinates $z = (q, p)$.

More compact notation:

$$\dot{z}^{\alpha} = J_c^{\alpha\beta} \frac{\partial H}{\partial z^{\beta}} = \{z^{\alpha}, H\}\,, \qquad \alpha, \beta = 1, 2, \ldots, 2N$$

Poisson matrix and bracket:

$$J_c = \begin{pmatrix} 0_N & I_N \\ -I_N & 0_N \end{pmatrix}, \qquad \{f, g\} = \frac{\partial f}{\partial z^{\beta}} J_c^{\alpha\beta} \frac{\partial g}{\partial z^{\beta}}$$

Canonical Transformation (CT)

- Coordinate change $z(\bar{z})$ or $\bar{z}(z)$ yields equations of motion

$$\dot{\bar{z}}^\alpha = \bar{J}^{\alpha\beta} \frac{\partial \bar{H}}{\partial \bar{z}^\beta}$$

- Hamiltonian transforms as a scalar:

$$\bar{H}(\bar{z}) = H(z)$$

- Poisson matrix J a contravariant 2-tensor (bivector)

$$\bar{J}^{\alpha\beta}(\bar{z}) = J_c{}^{\mu\nu} \frac{\partial \bar{z}^\alpha}{\partial z^\mu} \frac{\partial \bar{z}^\beta}{\partial z^\nu} = J_c{}^{\alpha\beta}\,, \qquad \alpha, \beta = 1, \ldots 2N$$

2nd equality $\Rightarrow$ CT or symplectomorphism $\Rightarrow \bar{z} = (\bar{q}, \bar{p})$

- Hamilton's Equations:

$$\dot{\bar{q}}^i = \frac{\partial \bar{H}}{\partial \bar{p}_i} \qquad \text{and} \qquad \dot{\bar{p}}^i = -\frac{\partial \bar{H}}{\partial \bar{q}_i}\,, \qquad i = 1, 2, \ldots N$$

Some Canonical Transformations

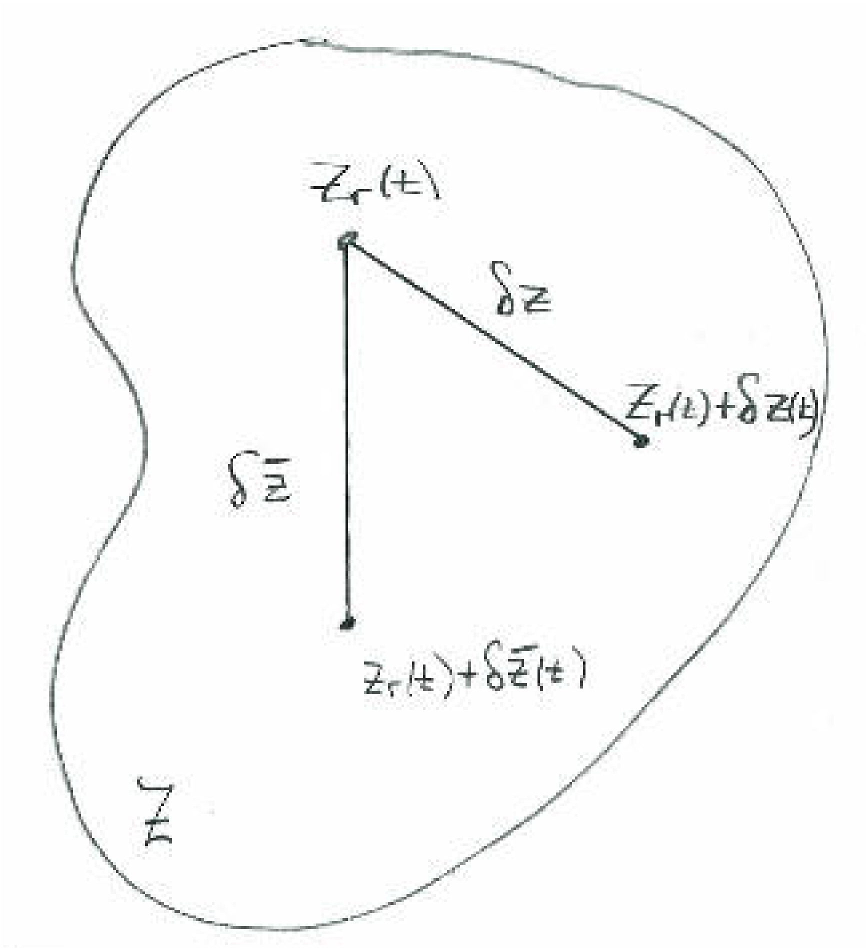
• Dynamics is a CT: for any time t, $z(z_0, t)$ which relates $z \leftrightarrow z_0$ is a CT. A special 1-parameter group of diffeomorphisms $g_t \colon \mathcal{Z} \to \mathcal{Z}$.

• Major theme: find CT that simplifies the dynamics $\to$ action-angle variables, perturbation theory, etc. For example

$$(q,p) \leftrightarrow (\theta, J) \quad \text{such that} \quad H(q,p) = \bar{H}(J)\,.$$

Properties of Hamiltonian Dynamics

- Three nearby trajectories: $z_r(t)$, $z_r(t) + \delta z(t)$, and $z_r(t) + \delta \bar{z}(t)$



Properties of Hamiltonian Dynamics (cont)

'Area':

$$\delta^2 A = \delta \bar{z}^{\alpha} \omega^c_{\alpha\beta} \delta z^{\beta}\,, \qquad \alpha, \beta = 1, 2, \ldots, 2N$$

$$\omega^c = \begin{pmatrix} 0_N & -I_N \\ I_N & 0_N \end{pmatrix} = (J_c)^{-1}$$

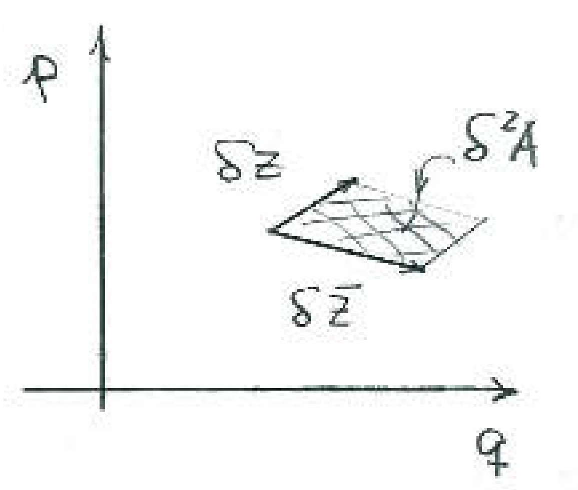
If Hamiltonian, then

$$\frac{d}{dt}\,\delta^2 A = 0$$

What is it?

What is it?

For $N = 1$, $\delta^2 A$ is the area of a parallelogram, i.e. $\delta^2 A = \delta\bar{p}\delta q - \delta\bar{q}\delta p = |\delta\bar{z} \times \delta z|$



For $N \neq 1$ this quantity is a sum over such areas (indexed by i),

$$\delta^2 A = \ \delta\bar{p}_i \delta q^i - \delta\bar{q}^i \delta p_i$$

and is called the first Poincaré invariant. Modern notation $\omega = dp_i \wedge dq^i$. Symplectic two-form etc.

Consequences

• Area of particular 2D ribbon is preserved.

• $2N$ Volume preservation $\Leftrightarrow$ Liouville's theorem.

• Everything in between, i.e., the Poincare invariants of dimension 2, 4, 6, ... 2N.

Loop integrals (circulation theorem):

$$\mathcal{J} = \oint_{\gamma} p \cdot dq \qquad \text{satisfy} \qquad \dot{\mathcal{J}} = 0\,,$$

for any closed curve γ in phase space. By a generalization of Stokes theorem, loop integrals and symplectic areas are related.

• If this were not true, sufaces of section of $\mathbf{B}$-lines would look very different.

Geometry

Hamiltonian dynamics $\Leftrightarrow$ flow on symplectic manifold

Phase space, $\mathcal{Z}$, a differential manifold endowed with a closed, nondegenerate 2-form ω (recall $\delta^2 A$)

Poisson bivector is inverse of ω.

Flows generated by Hamiltonian vector fields $Z_H = JdH$, H a 0-form, dH a 1-form. Poisson bracket = commutator of Hamiltonian vector fields etc.

Early references: Jost, Mackey, Souriau, Abraham, . . .

Noncanonical Hamiltonian Dynamics

Sophus Lie (1890)

Noncanonical Coordinates:

$$\dot{z}^a = J^{ab}\frac{\partial H}{\partial z^b} = \{z^a, H\}\,, \qquad \{f,g\} = \frac{\partial f}{\partial z^a}J^{ab}(z)\frac{\partial g}{\partial z^b}\,, \qquad a,b = 1,2,\ldots M$$

Poisson Bracket Properties:

antisymmetry $\longrightarrow$ $\{f,g\} = -\{g,f\}\,,$

Jacobi identity $\longrightarrow$ $\{f,\{g,h\}\} + \{g,\{h,f\}\} + \{h,\{f,g\}\} = 0$

G. Darboux: $det J \neq 0 \Longrightarrow J \rightarrow J_c$ Canonical Coordinates

Sophus Lie: $det J = 0 \Longrightarrow$ Canonical Coordinates plus Casimirs

$$J \rightarrow J_d = \begin{pmatrix} 0_N & I_N & 0 \\ -I_N & 0_N & 0 \\ 0 & 0 & 0_{M-2N} \end{pmatrix}.$$

Flow on Poisson Manifold

Definition. A Poisson manifold $\mathcal{M}$ is differentiable manifold with bracket $\{\,,\,\} : C^\infty(\mathcal{M}) \times C^\infty(\mathcal{M}) \to C^\infty(\mathcal{M})$ st $C^\infty(\mathcal{M})$ with $\{\,,\,\}$ is a Lie algebra realization, i.e., is i) bilinear, ii) antisymmetric, iii) Jacobi, and iv) consider only Leibniz, i.e., acts as a derivation.

Flows are integral curves of noncanonical Hamiltonian vector fields, $Z_H = JdH$.

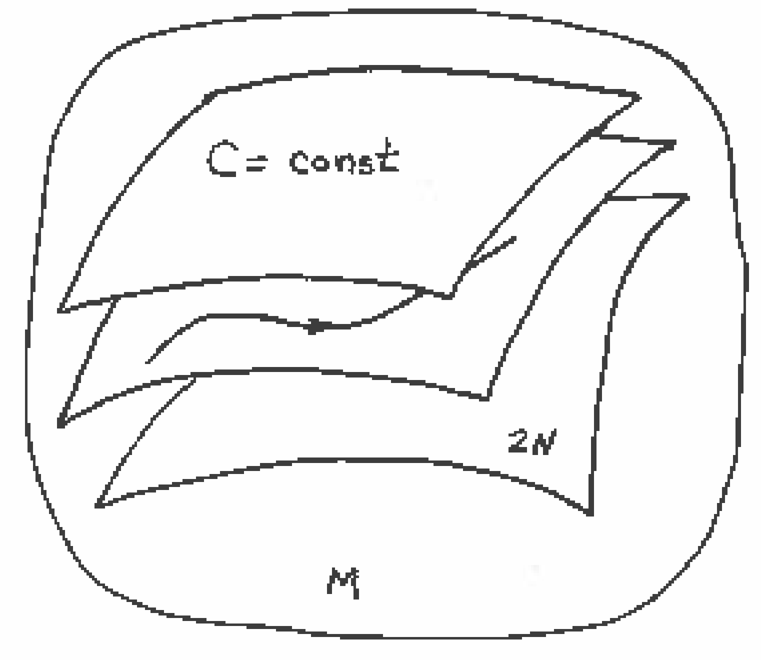
Because of degeneracy, $\exists$ functions C st $\{f, C\} = 0$ for all $f \in C^\infty(\mathcal{M})$. Called Casimir invariants (Lie's distinguished functions.)

Poisson Manifold $\mathcal{M}$ Cartoon

Degeneracy in $J \;\Rightarrow$ Casimirs:

$$\{f, C\} = 0 \quad \forall\ f : \mathcal{M} \to \mathbb{R}$$

Lie-Darboux Foliation by Casimir (symplectic) leaves:



Leaf vector fields, $Z_f = \{z, f\} = Jdf$ are tangent to leaves.

Lie-Poisson Brackets (cf. Yoshida and Hirota talks)

Matter models in Eulerian variables:

$$J^{ab} = c^{ab}_{c} z^{c}$$

where c^{ab}_{c} are the structure constants for some Lie algebra.

Examples:

• 3-dimensional Bianchi algebras for free rigid body, Kida vortex, rattleback

• Infinite-dimensional theories: Ideal fluid flow, MHD, shearflow, extended MHD, Vlasov-Maxwell, etc.

Integrability vs. Chaos

Integrability

Definition. An N degree-of-freedom Hamiltonian systems is integrable in the sense of Liouville if there exist N constants of motion, $I_i(z)$, $i = 1, 2, \ldots, N$, that are smooth (or analytic), independent, single-valued, and in involution.

Loosely speaking the set $\mathcal{S} = \{z | I_i = \mathring{I}_i \;\; \forall i = 1, 2, \ldots, N\}$ defines an N-dimensional invariant submanifold of $\mathcal{Z}$ and involution,

$$\{I_i, I_j\} = 0\,,$$

implies these can be used as new momenta and that $\mathcal{S}$ is parallellizable, i.e., it is of dimension N and admits N smooth linearly independent (Hamiltonian) vector fields at all points $z \in \mathcal{S}$.

Theorem *A compact N-dimensional parallellizable manifold with N commuting vector fields is an N-torus.*

Action-Angle Variables

(Jost-Arnold) Given the above there is coordinate change to action-angle variables

$$(q, p) \leftrightarrow (\theta, J)$$

and $\mathcal{Z}$ is foliated by N-tori, i.e. $\mathcal{S} = \mathbb{T}^N$.

The solution can be written down in these coordinates

$$\dot{\theta}^i = \Omega^i(J) = \frac{\partial \bar{H}(J)}{\partial J_i} \quad \text{and} \quad \dot{J}_i = -\frac{\partial \bar{H}(J)}{\partial \theta^i} = 0$$

implying

$$\theta = \Omega(J)t + \mathring{\theta} \quad \text{and} \quad J = \mathring{J}$$

However, Poincaré and Siegel theorem says integrable systems are measure zero in the set of all Hamiltonian systems.

Hamiltonian chaos is the lack of integrability. So, what happens in a typical 'chaotic' Hamiltonian system?

Destruction of Tori

Flows and Maps

Moser:

interpolation theorem $\Rightarrow$ associated with every area preserving (symplectic) map is a smooth Hamiltonian flow (set of ode's)

Poincare:

Bounded two degree-of-freedom systems = area preserving maps

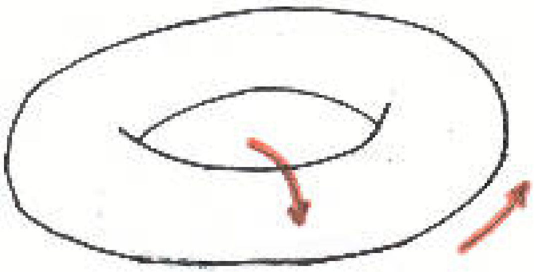
$$\dot{q}^i = \frac{\partial H(q,p)}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial H(q,p)}{\partial q^i} \qquad \text{for} \quad i = 1,2$$

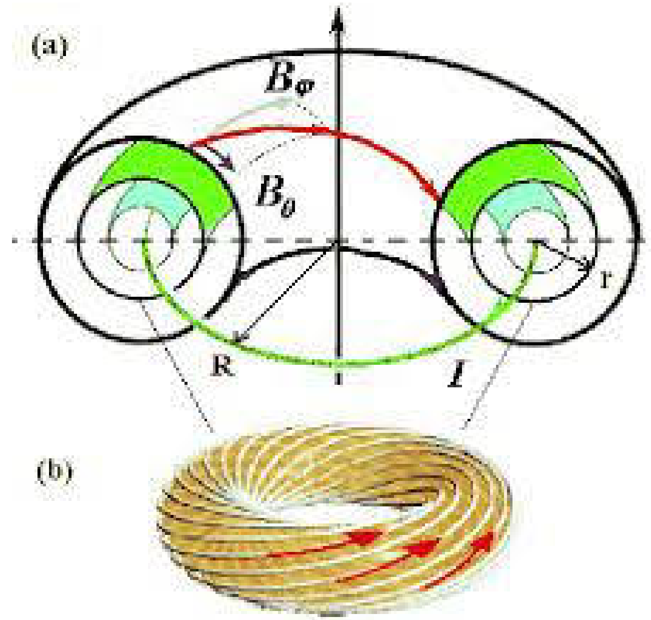
$$\Longleftrightarrow$$

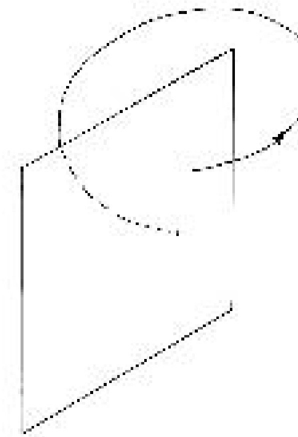
$$T : \mathbb{T} \times \mathbb{R} \to \mathbb{T} \times \mathbb{R} \qquad \text{s.t.} \qquad T(|\text{set}|) = |\text{set}|$$

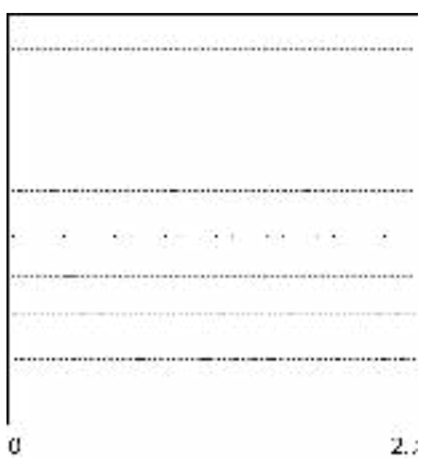
$$(x,y) \mapsto (x', y')$$

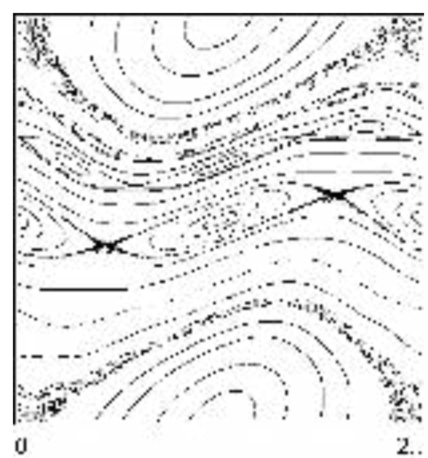
Poincaré's Surface of Section

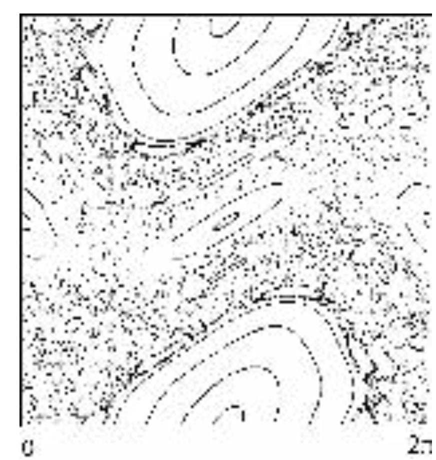












Symmetry breaking $\Leftrightarrow$ $k \uparrow$

Universal Symplectic Maps of Dimension Two

Standard (Twist) Map:

$$\begin{aligned} x' &= x + y' \\ y' &= y - \frac{k}{2\pi}\sin(2\pi x) \end{aligned}$$

Standard Nontwist Map:

$$\begin{aligned} x' &= x + a(1 - y'^2) \\ y' &= y - b\sin(2\pi x) \end{aligned}$$

Parameters:

a measures shear, while b and k measure ripple

Torus Breakup: 2 DoF Results

$\exists$ Action-Angle Variables: $H(q_1, q_2, p_1, p_2) \to H(J_1, J_2)$

ϕ_1, ϕ_2 ignorable $\Rightarrow$ foliation by tori

e.g. field lines of tokamak equilibrium

$\nexists$ Action-Angle Variables: (broken tori almost always the case)

- KAM theorem $\to$ applies near integrable

Rigorous and interesting, but was expected.

- Greene's method $\to$ works near and far from integrable

Partially rigorous, but physically more important than KAM (last barrier to transport). Conceptually more interesting (describes local behavior).

Greene's Idea

The sudden change from stability to instability of high order nearby periodic orbits is coincident with the breakup of invariant tori.

Example:

For which k-value of standard map is the (last) torus with *rotation number* $\omega^* = 1/\gamma$, the inverse golden mean, critical?

Rotation Number:

$\omega := \lim_{n\to\infty} \frac{x_n}{n}$ lifted to $\mathbb{R}$ $\quad$ $q-$profile $\sim \omega^{-1}$

Extensions:

$\omega^* :=$ quadratic irrational, e.g. $1/\gamma, 1/\gamma^2$, noble numbers, numbers with periodic continued fraction expansions, ...

Greene's Method

1. 'Approxmate' invariant torus (far from KAM limit) by sequence of periodic orbits with rotation numbers

$$\omega_i = \frac{n_i}{m_i}, \qquad n_i, m_i \in \mathbb{Z}$$

such that

$$\lim_{i \to \infty} \omega_i = \omega^*$$

Golden Mean Example: $1/\gamma = [0, 1, 1, 1 \ldots]$,

where $\gamma = (\sqrt{5} + 1)/2$, with convergents

$$a_i = \frac{F_i}{F_{i+1}}, \qquad F_i, F_{i+1} \in \mathbb{Z}$$

where F_i are the Fibonacci numbers, which are truncations of the inverse golden mean continued fraction expansion

Higher and higher order $\longrightarrow$

looks more and more like the $1/\gamma$-invariant torus

Greene's Method (Continued)

2. Calculate 'Residues'

$$R := \frac{1}{4}\left[2 - \text{trace} DT^n\right]$$

for sequence of periodic orbits and consider

$$\lim R_i = \begin{cases} 0 & \text{torus exists} \\ \infty & \text{torus does not exist} \\ R_c \sim .25 & \text{torus critical} \end{cases}$$

For the standard map Greene calculated

$$k_c = .971635\ldots$$

for criticality of the $1/\gamma$-torus, the last torus.

How did he do it? Need periodic orbits. How many?

Used involution decomposition to obtain periodic orbits $\sim 10^6$.

Involution Decomposition

Birkhoff, de Voglaere, Greene

Discrete Symmetries (e.g. time reversal) $\Longrightarrow$

$$T = I_1 \circ I_2 \,,$$

where

$$I_1 \circ I_1 = I_2 \circ I_2 = \text{identity map}$$

Reduces 2-dimensional root search to a 1-dimensional search along *symmetry sets*.

• Enables one to obtain periodic orbits of order 10^8 with 13 place accuracy!

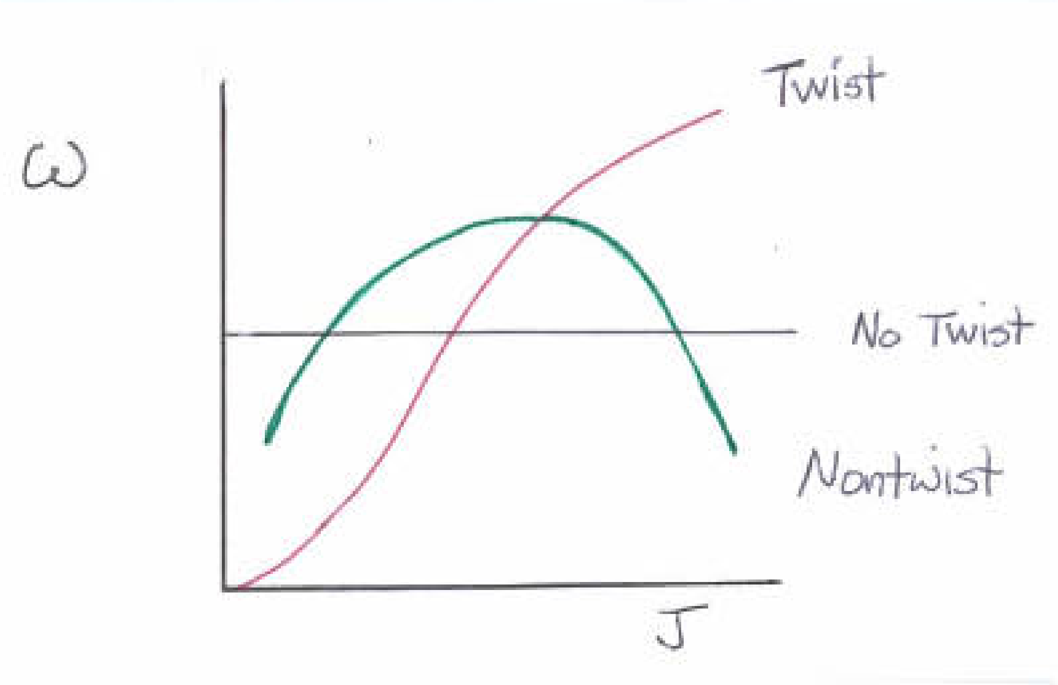
Chaos in Nontwist Hamiltonian Systems

Collaborators: J. M. Greene, D. del-Castillo-Negrete, A. Wurm, A. Apte, K. Fuchss, I. Caldas, R. Viana, J. Szezech, Lopes, ...

http://www.scholarpedia.org/article/Nontwist_maps

What is nontwist?

<u>Twist</u> (Moser): image of vertical line under symplectic (area preserving) map of cylinder $S \times \mathbb{R}$ has property of 'further up $\Rightarrow$ further over. Poincare-Birkhoff, early KAM, Aubry-Mather, ...



Whence twist?

Natural Hamiltonians:

$$H = p^2/2 + V(q) \longrightarrow \dot{q} = p$$

Integrable systems:

$$H = H(J) \longrightarrow \omega(J) = \partial\omega/\partial J$$

nontwist = 'generic' way twist condition is violated

- structurally stable -

Universal Nontwist Map

Standard Nontwist Map:

$$
\begin{aligned}
x' &= x + a(1 - y'^2) \\
y' &= y - b\sin(2\pi x)
\end{aligned}
$$

Parameters:

a measures shear, b ripple

Shearless Curve:

$$\text{for } b = 0\,, \qquad \frac{\partial x'}{\partial y} - -2ay' = 0 \Rightarrow y = 0$$

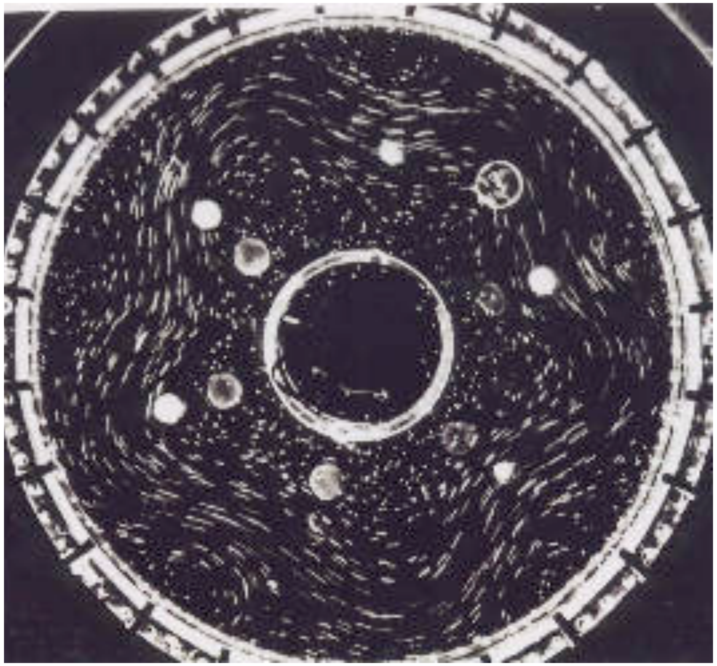
Does it occur in physical systems?

Yes, many!

Swinney's Rotating Tank (circa 1987)

Cyclonic (eastward) jet

particle streaks



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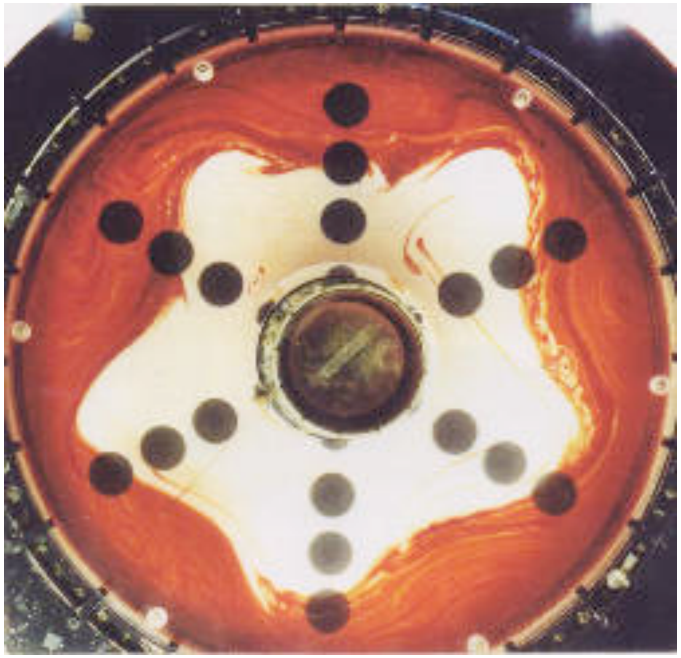
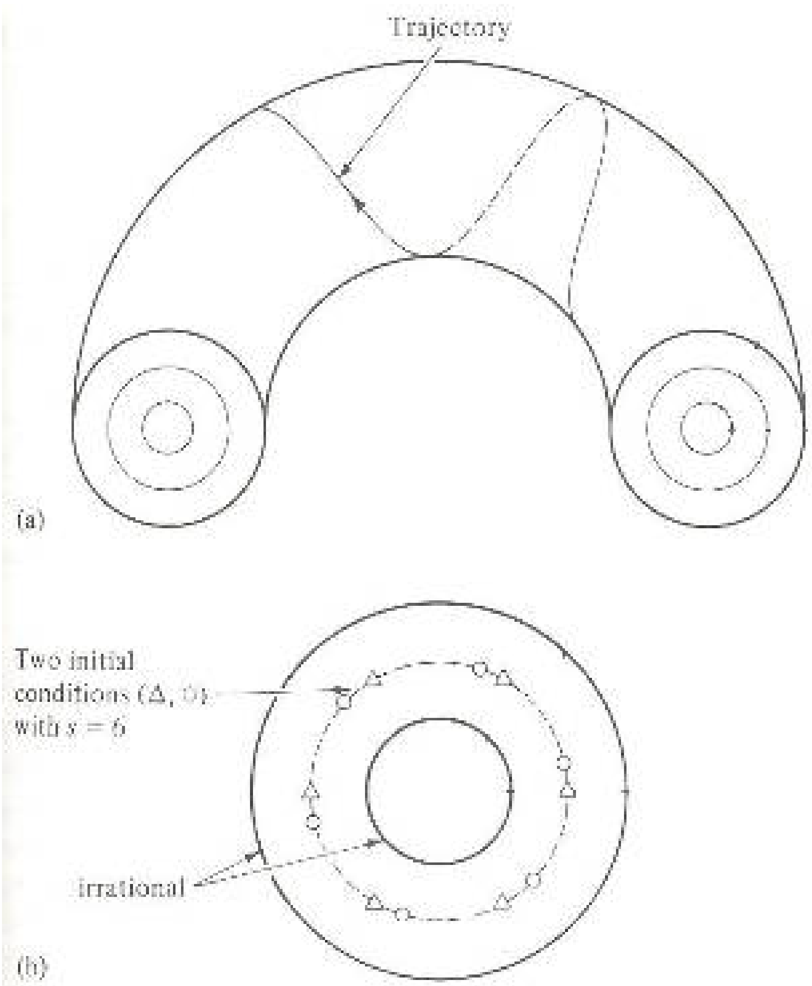


Applications
(discovery, rediscovery, re-rediscovery)

- RF in ptle accelerators (Symon and Sessler, 1956)
- Keplerian orbital corrections due to oblateness (Kyner, 1968)
- Laser-plasma coupling (Langdon and Lasinsky, 1975)
- Magnetic fields lines for double tearing mode (Stix, 1976)
- Wave-particle interactions (Karney, 1978; Howard et al., 1986)
- Storage ring beam-beam interaction (Gerasimov et al., 1986)
- Transport and mixing in traveling waves (Weiss, 1991)
- Ray propagation in waveguides with lenses (Abdullaev, 1994)
- SQUIDs (Kaufman et al., 1996)
- Relativistic oscillators (Kim, Lee, 1995; Luchinsky ..., 1996)
- B-lines in stellerators (Davidson ..., 1995; Hayashi ..., 1995)
- $E \times B$ transport (Horton ..., 1998; del-Castillo-Negrete, 2000)
- Circular billiards (Kamphorst and de Carvalho, 1999)
- Self-consistent transport (del-Castillo-Negrete ..., 2002)
- Atomic physics (Chandre et al., 2002)
- Stellar pulsations (Munteanu et al., 2002)

B-lines in Plasma Fusion Devices



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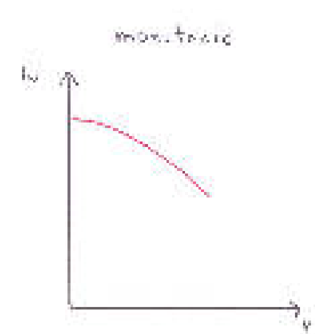
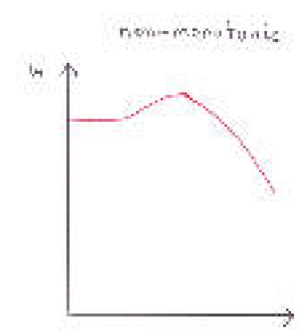
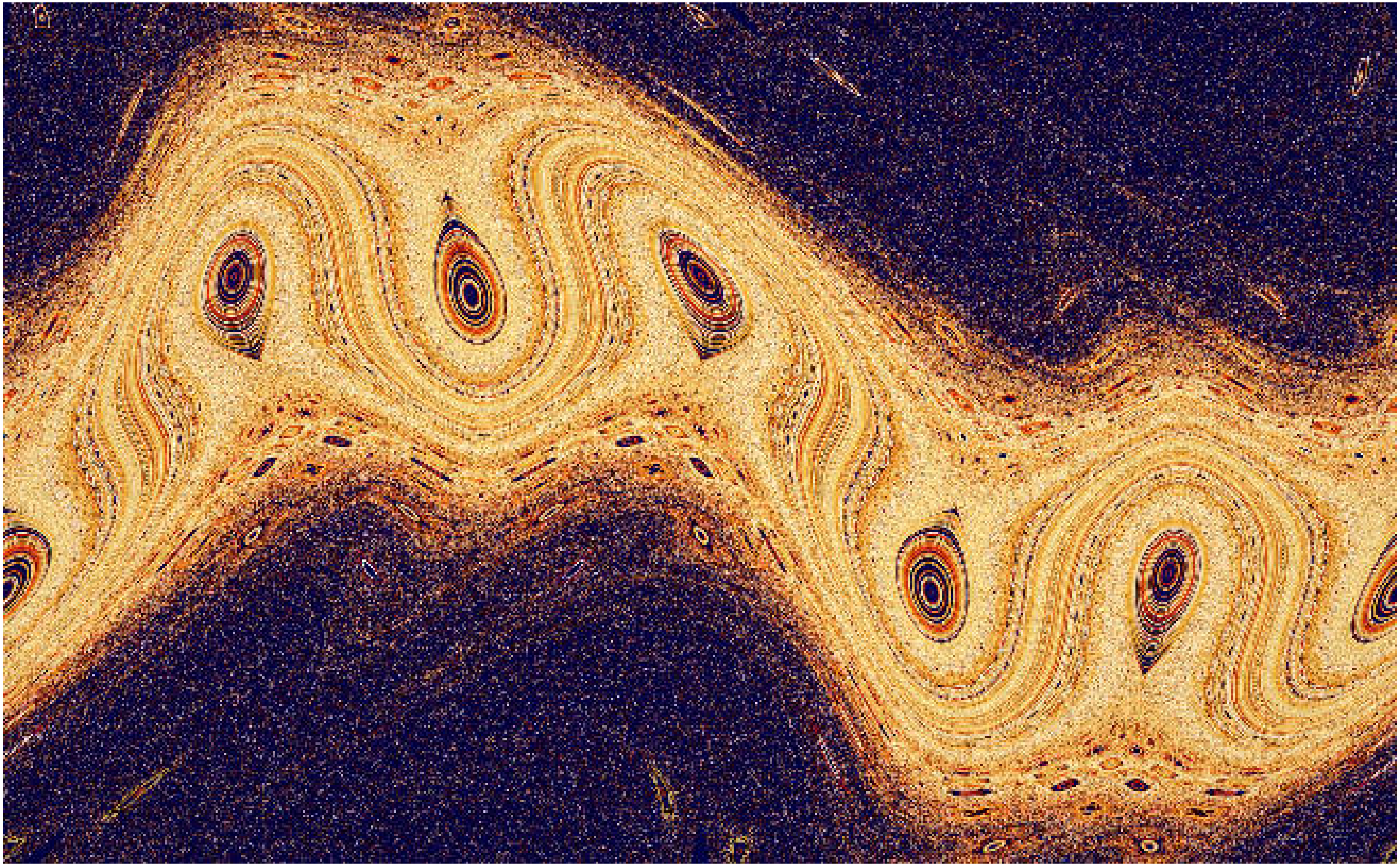
Examples of winding number profiles:



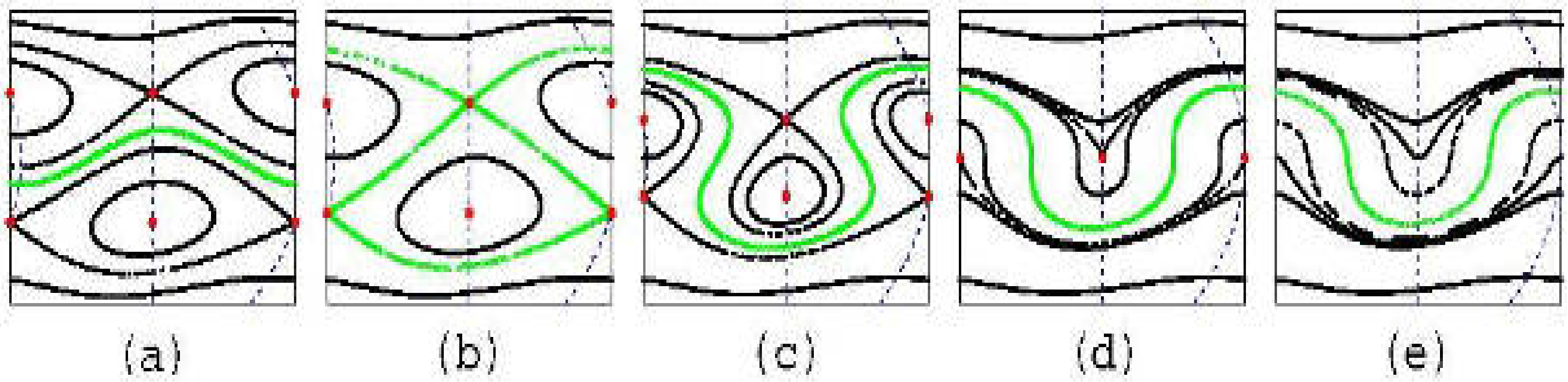


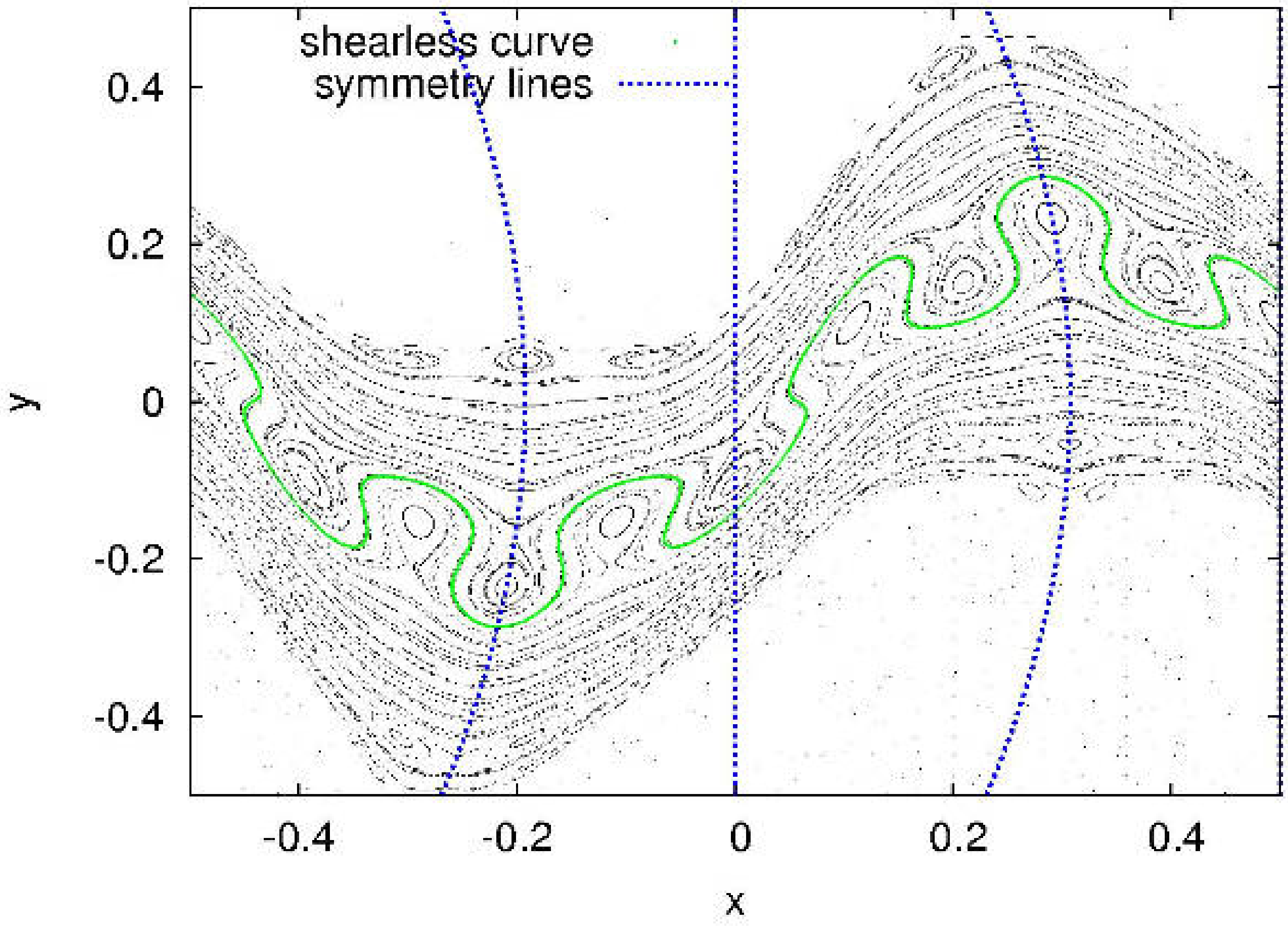
What difference does nontwist make?



— George Miloshevic

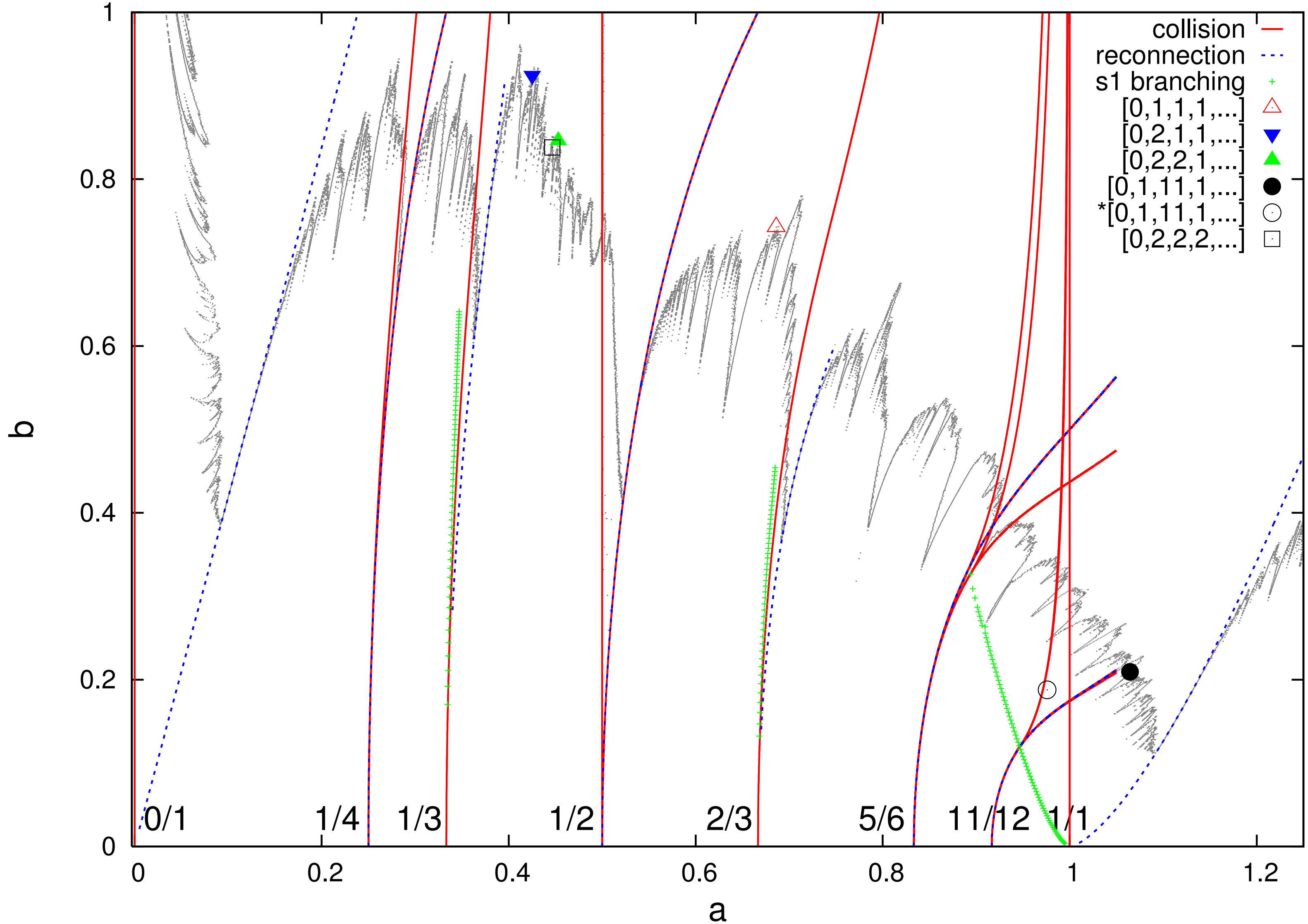
Reconnection – Different Island Structures




shearless curve
symmetry lines
0.4
0.2
0
-0.2
-0.4
y
-0.4
-0.2
0
0.2
0.4
x

Torus Destruction Differs. How?

Brute Force



collision
reconnection
s1 branching
[0,1,1,1,...]
[0,2,1,1,...]
[0,2,2,1,...]
[0,1,11,1,...]
*[0,1,11,1,...]
[0,2,2,2,...]
0/1
1/4
1/3
1/2
2/3
5/6
11/12
1/1
b
a
0
0.2
0.4
0.6
0.8
1
1.2

Nontwist Difficulties:

Codimension two, a, b.

Which periodic orbits should be used? Complicated parameter dependent nested homoclinic and heteroclinic connections with changing topologies at finer and finer scales.

Periodic orbits do not exist for all convergents! They sometimes occur in pairs, but sometimes these pairs have collided and vanis-hed depending on parameters.

Which torus will be the last to go? Noble? Shearless? Parameter values to use? Path in parameter space?

How to know?

Shearless curve of standard nontwist map:

For $b = 0$ obvious, for $b \neq 0 \longrightarrow$

r/s-Bifurcation Curve: locus of points (a, b) s.t. r/s periodic orbits are at collision point:

$$b = \Phi_{r/s}(a)$$

- *Shearless periodic orbits* are on the r/s-bifurcation curve.

- Shearless periodic orbits limit to *shearless irrational invariant tori*.

- Codimension two: chose ω_* and shearlessness.

For (a, b) s.t. $b = \Phi_{r/s}(a)$ $\exists$ half of the Fibonacci sequence that limits to $1/\gamma$.

Adaptation of Greene - Nontwist (Shearless) Results

• We first found (a, b) for $1/\gamma$ by 'intelligent search' such that residue limit to a period-6 cycle at criticality

$$\{R_1^*, R_2^*, \ldots, R_6^*\}$$

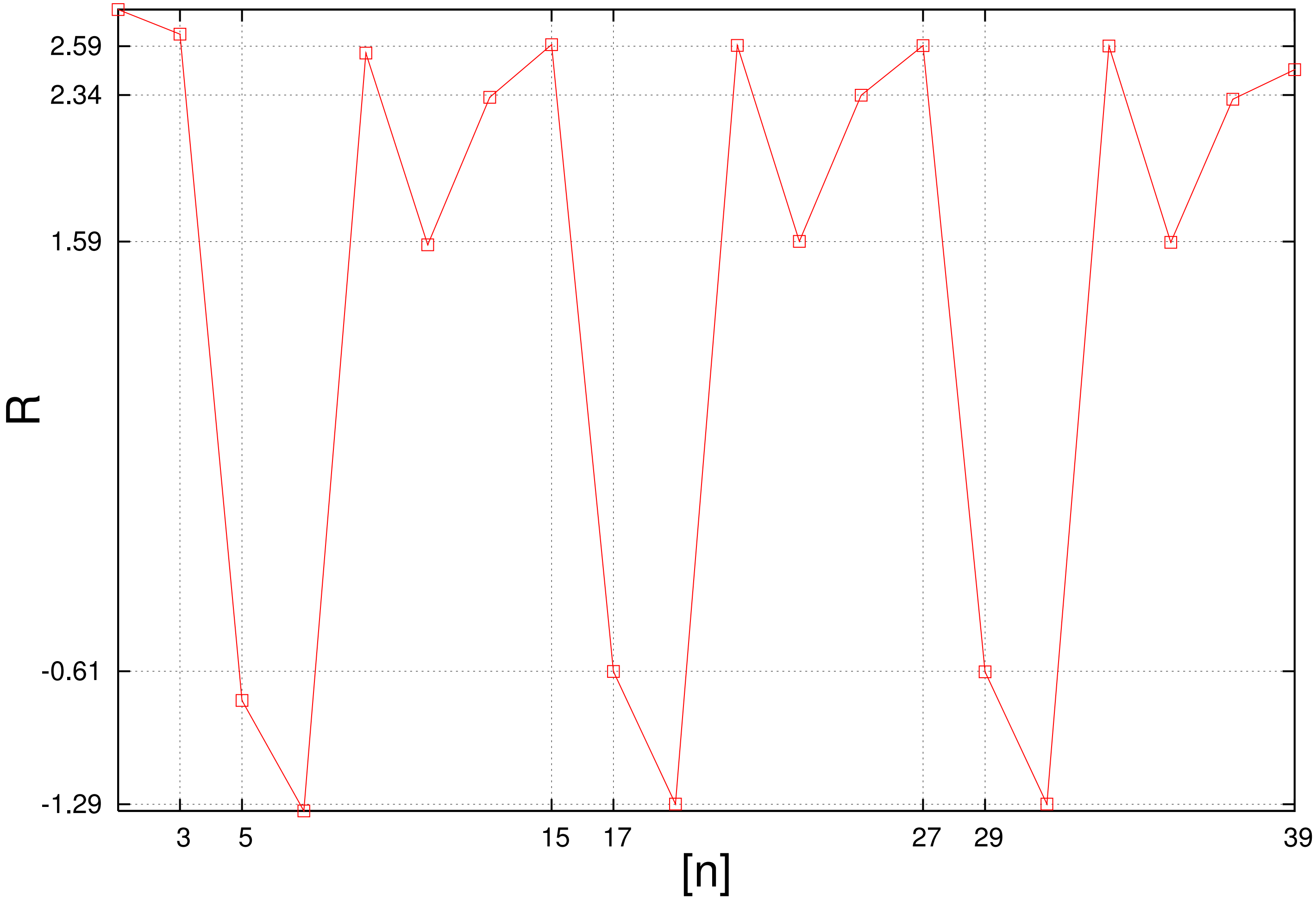
That is, there exist six convergent subsequences.

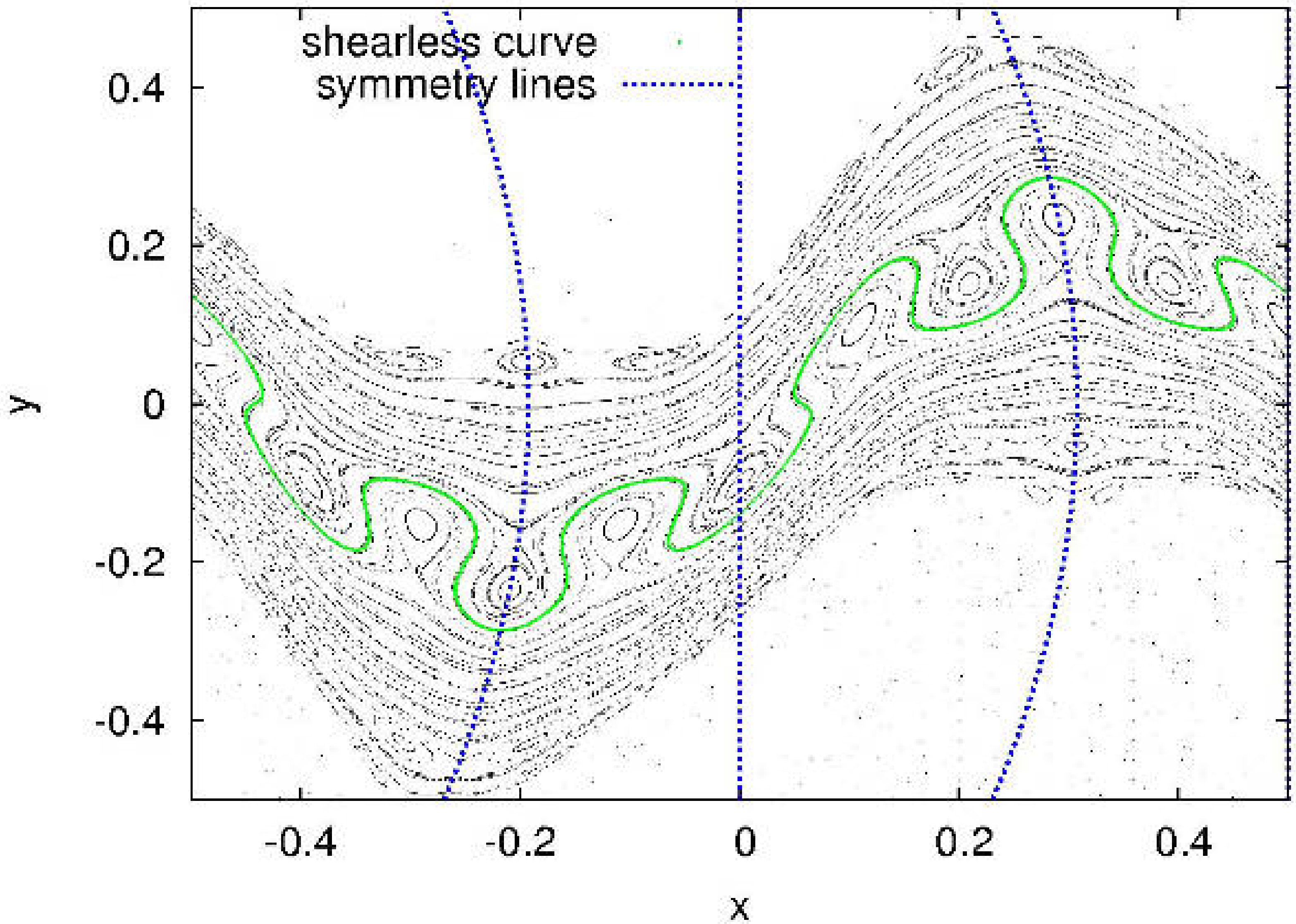
Doing this for periodic orbits of order 10^6 (15 years ago)

$$a \approx 0.686049 \qquad b \approx 0.742497002412$$

• Subsequently, many results related to other shearless tori. Recently for $[2, 2, 2, 2, ...]$ a new kind of breakup.

Period-6 Residue 'Dynamics'





shearless curve
symmetry lines
0.4
0.2
0
-0.2
-0.4
y
-0.4
-0.2
0
0.2
0.4
x

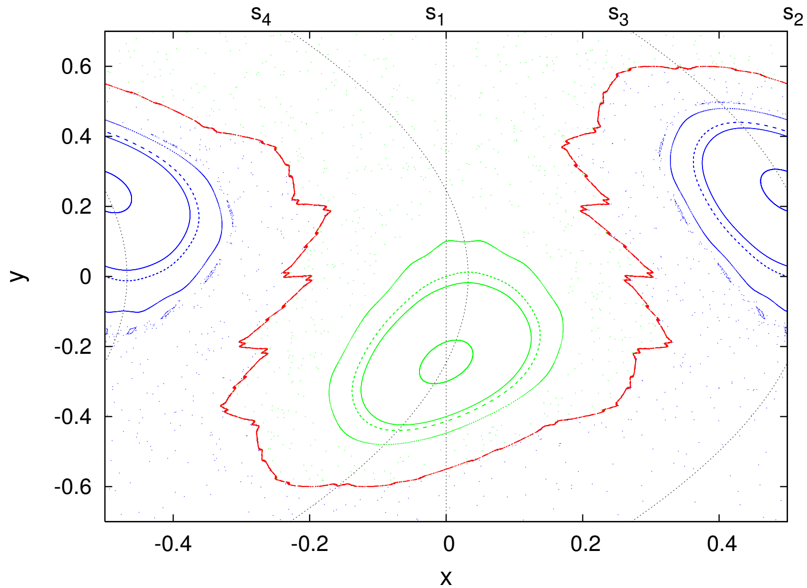
Renormalization

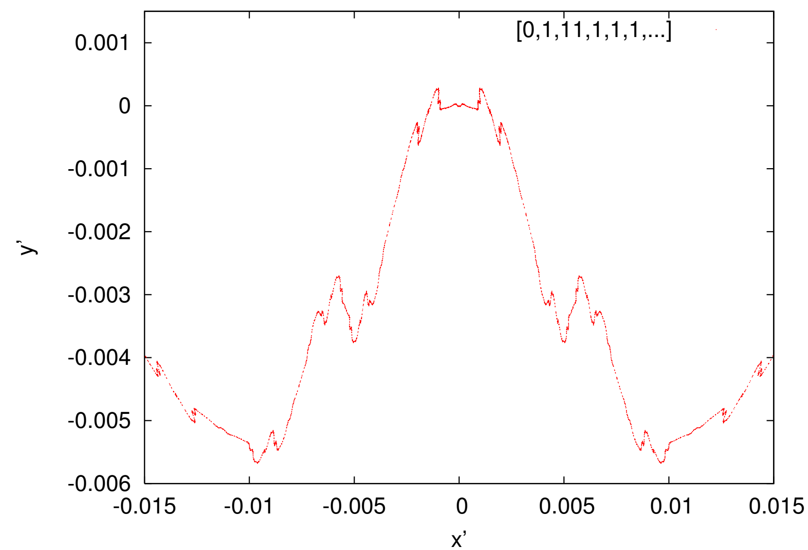
Greene (1968,1979)$\rightarrow$ MacKay (1980) $\rightarrow$
Del-Castillo-Negrete et al. (1995)

Like a phase transition: universal exponents, scaling, etc.

e.g. zoom area by $\sim 10^4 \longrightarrow$

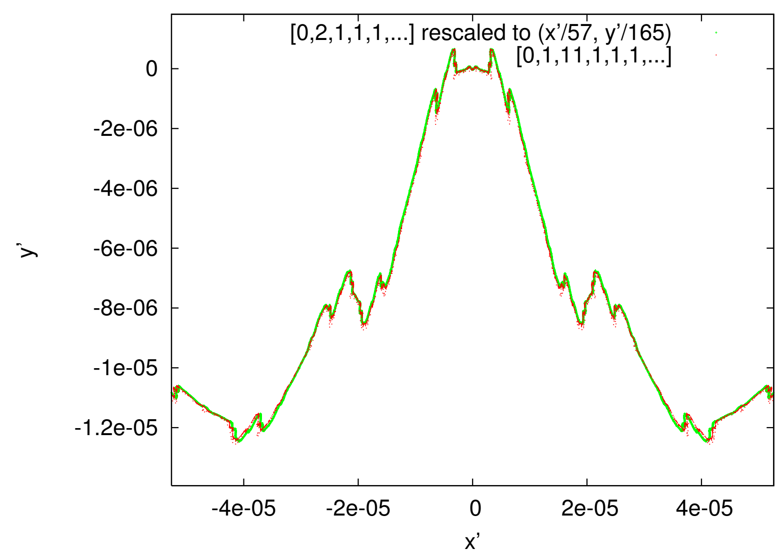
Renormalization: Scaling Invariance





Scaling invariance:

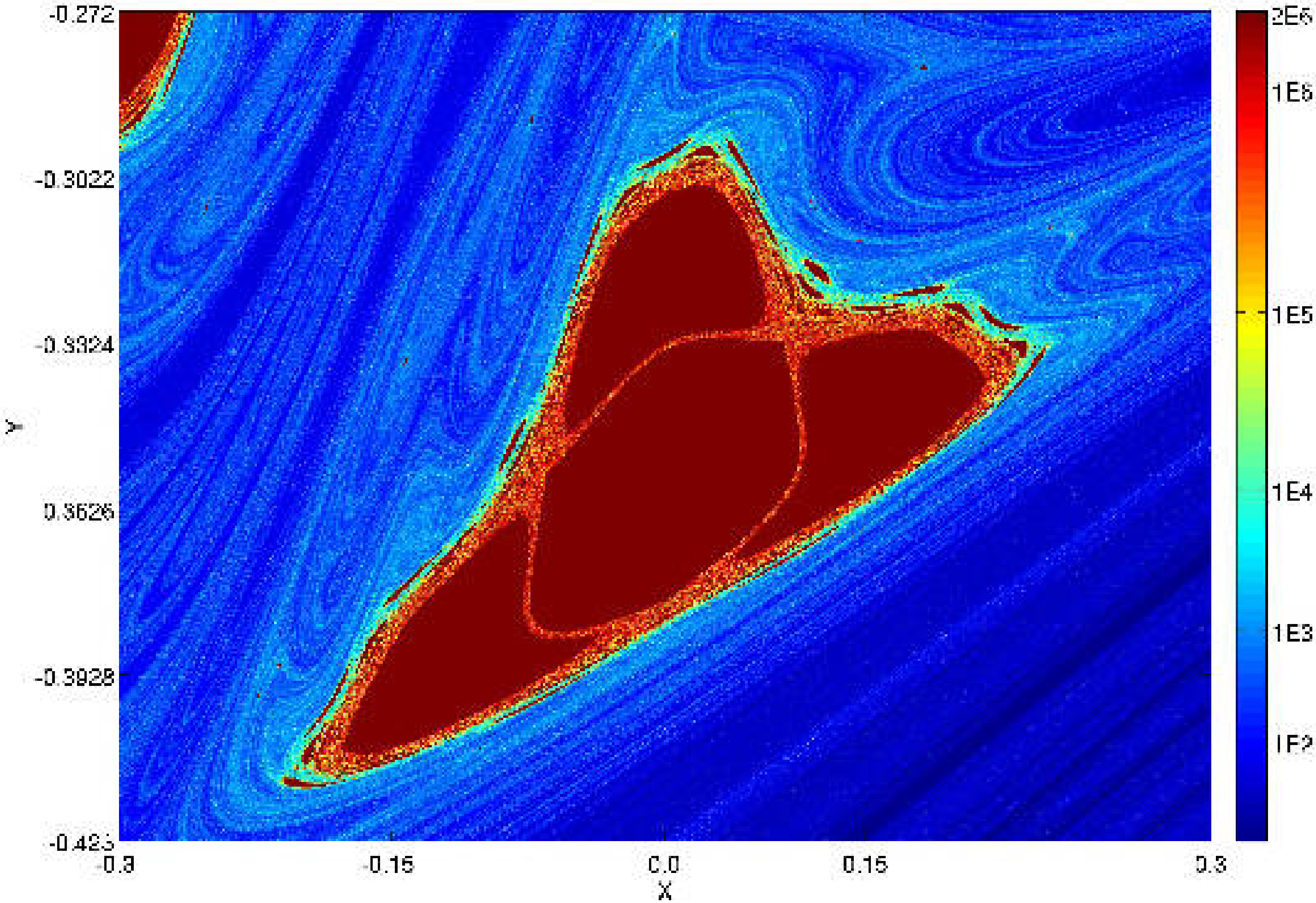
$$(x, y) \to (\alpha x, \beta y)$$



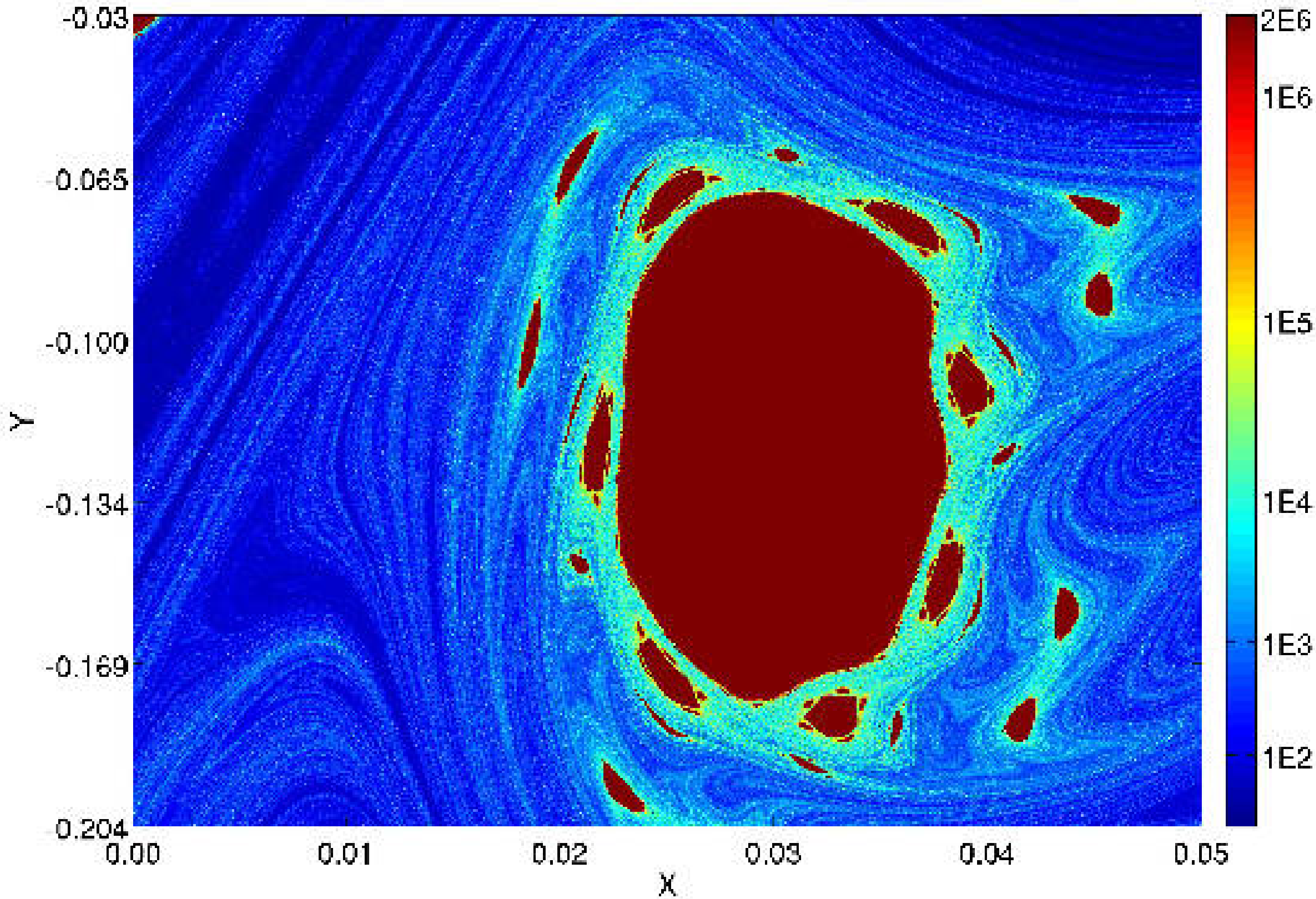
Numerical estimate:

$$\alpha = 321.65 \pm 0.070$$
$$\beta = 431.29 \pm 0.19$$

Stickiness of Nontwist Systems



-0.272
-0.3022
-0.3324
-0.3626
-0.3928
-0.423
Y
-0.3
-0.15
0.0
0.15
0.3
X
2E6
1E6
1E5
1E4
1E3
1E2



-0.03
-0.065
-0.100
-0.134
-0.169
-0.204
Y
0.00
0.01
0.02
0.03
0.04
0.05
X
2E6
1E6
1E5
1E4
1E3
1E2