

# Self-organized confinement by magnetic dipole: recent results from RT-1 and theoretical modeling

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**Abstract.** Inhomogeneous magnetic field gives rise to interesting properties of plasmas which are degenerate in homogeneous (or zero) magnetic fields. Magnetospheric plasmas, as observed commonly in the Universe, are the most simple, natural realization of strongly inhomogeneous structures created spontaneously in the vicinity of magnetic dipoles. The RT-1 device produces a “laboratory magnetosphere” by which stable confinement (particle and energy confinement time  $\sim 0.5$  s) of high- $\beta$  (local electron  $\beta \sim 0.7$ ; electron temperature  $\gtrsim 10$  keV) plasma is achieved. By producing a pure-electron plasma, we obtain clear-cut evidence of inward (or up-hill) diffusion of particles. A statistical mechanical model reveals the “distortion” of phase space, induced by the inhomogeneity of the ambient magnetic field, on which the plasma relaxes into an equilibrium with inhomogeneous density while it maximizes the entropy.

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## 1. Introduction

Magnetospheres are self-organized structures found commonly in the Universe. The strong inhomogeneity of a dipole magnetic field gives rise to interesting properties of plasmas which are degenerate in homogeneous (or zero) magnetic fields. We observe spontaneous creation of a steep density gradient towards the higher-magnetic field core region. This naturally produced *confinement* may be applied to various technologies such as advanced fusion[1] and antimatter physics[2]. Apart from these practical purposes, understanding the mechanism that operates beneath this interesting phenomenon is a profound problem of physics study, which possibly provides us with a new perspective of describing the physics of plasmas.

The aim of this article is to present an interim summary of the experimental observations and theoretical modeling of the self-organized magnetospheric plasmas. We have constructed a “laboratory magnetosphere,” the RT-1 device [3] by which we have demonstrated stable confinement (particle and energy confinement times  $\sim 0.5$  s) of very high- $\beta$  plasma (local electron beta  $\sim 0.7$ ) [4, 5]. Using RT-1, we can also produce a pure-electron non-neutral plasma by injecting electrons from an electron gun placed at the periphery of the confinement region (bounded by a magnetic separatrix). Probing the internal electric field, we have found a clear-cut evidence of up-hill diffusion of electrons [6, 7]. Based on these experiments, we postulate that the self-organization is a spontaneous process to vanish a “free energy” of collective motion by relaxing into an equilibrium state on a *macroscopic scale hierarchy*. A theoretical explanation for the self-organized confinement is given by constructing a distribution function on such a macroscopic scale hierarchy.

This paper is organized as follows: In section 2, we describe the main parameters of the RT-1 device and the experimental observation of “spontaneous confinement.” In section 3, we present a theoretical model of the self-organization.

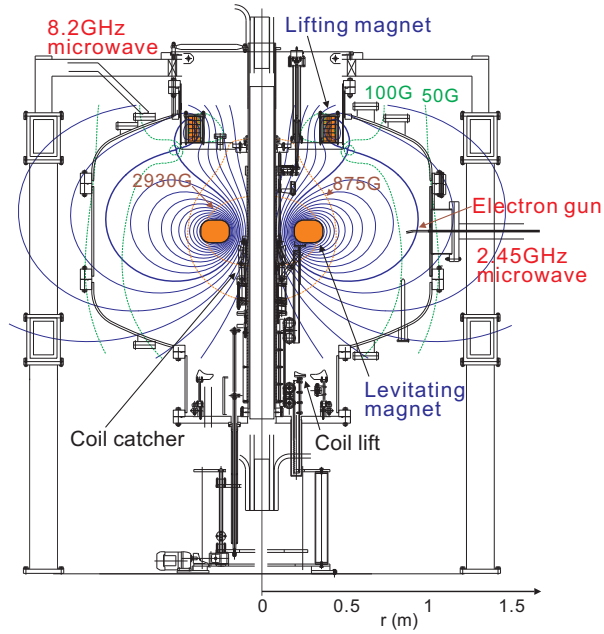
## 2. Laboratory magnetosphere produced in RT-1

The RT-1 device is a laboratory magnetosphere; levitating a superconducting (Bi-2223 high- $T_c$  superconductor) ring magnet (0.25 MA) in a vacuum chamber (diameter  $\sim 2$  m), we can produce a magnetospheric plasma (see figure 1). The field strength in the confinement region varies from 0.5 T to 0.01 T. The plasma is bounded by a magnetic separatrix that is produced by the lifting magnetic field.

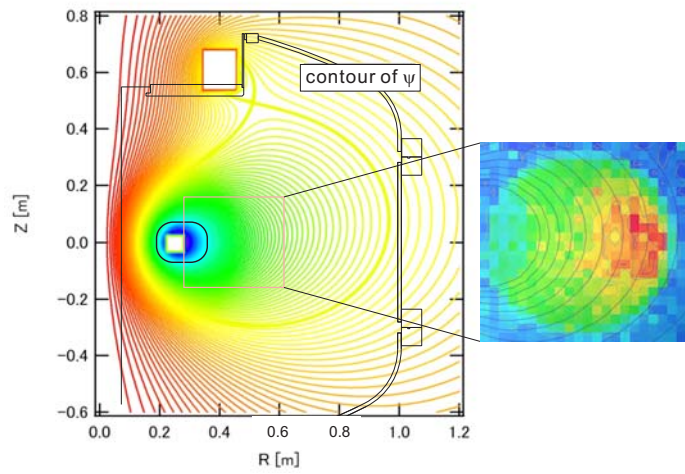
### 2.1. High- $\beta$ plasma

High-temperature plasma is produced by ECH (8.2 GHz, 25 kW and 2.45 GHz, 20 kW). Electrons consist of hot and cold components; the temperature of the hot component is 10  $\sim$  50 keV, while that of the cold component is typically  $\sim 100$  eV. The total electron density is of order  $10^{17}$  m $^{-3}$ . When the filling gas pressure is low ( $\lesssim 10^{-3}$  Pa), the cold component is less than 30% of the total electrons [5]. Because of low density and high electron temperature, ions are left cold ( $\lesssim 10$  eV); direct ion heating by ICH is the main subject of the on-going Phase-II project [8].

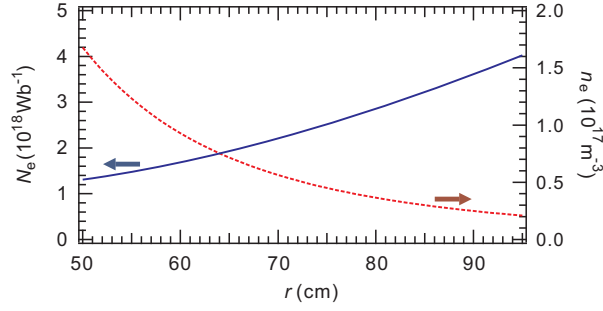
By matching four-point diamagnetic flux data (as well as inserted magnetic probe data from benchmark experiment) with solutions of the Grad-Shafranov equation, we estimate the pressure profile in the plasma; see figure 2. High- $\beta$  (local  $\beta \sim 0.7$ ) stable confinement has been demonstrated with simultaneous high electron temperature



**Figure 1.** Schematic drawing of the RT-1 device. A dipole magnetic field is produced by the levitating superconducting magnet. A high- $\beta$  plasma is produced and heated by ECH (8.2 GHz and 2.45 GHz). A pure-electron non-neutral plasma is also produced by injecting electrons from an electron gun placed at the periphery of the confinement region (bounded by a magnetic separatrix).



**Figure 2.** Left: High- $\beta$  equilibrium reconstructed by a solution of the Grad-Schafranov equation fitting diamagnetic signals. Right: Soft-x ray image of the plasma (by auxiliary lines, we show the geometric relation between two plots, but they are from different plasma shots).



**Figure 3.** The radial profiles of the electron density  $n_e$ , and the electron number  $N_e$  per unit flux-tube volume  $\oint dl/B$  (for the original data, see figure 8 of [5]).

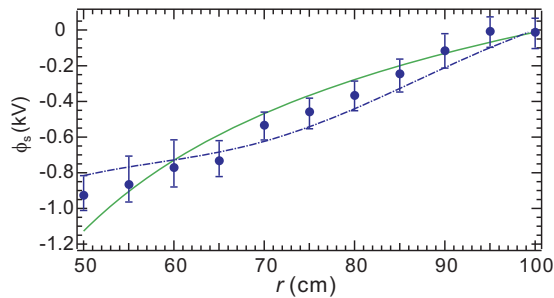
and long confinement time (particle and energy confinement times  $\sim 0.5$  s, which are estimated by the free-decay times after stopping the ECH). The estimated plasma pressure is consistent with the electron temperature and density [4, 5]. The confinement times can be accounted for by classical atomic processes: the ion confinement time is of the same order of the charge-exchange time of ions ( $\gtrsim 0.1$  s). While hot electrons are decoupled with ions, they couple with the cold electrons with energy equilibration time ( $\sim 1$  s), and cold electrons are lost with ions by the ambipolar mechanism.

The radial profile of the electron density  $n_e(r)$  (measured by 75 GHz microwave interferometry) is highly peaked (giving an indirect proof of the *inward diffusion* of particles [9, 10]; in subsection 2.2, we give a clear-cut evidence of inward diffusion in a pure-electron non-neutral plasma; the reader is also referred to similar observation in a different experiment [11], as well as discussions on astronomical magnetospheres [12, 13]). Fitting the data by a function  $n_e(r) = n_0 r^{-\alpha}$ , we estimate  $\alpha = 2.8 \pm 0.4$  for a wide range of operating parameters [5]. Multiplying  $n_e$  by  $\oint dl/B$  ( $dl$  is the parallel unit length along the magnetic surface), we can estimate the particle number  $N_e$  in a unit magnetic-flux tube; see figure 3. While  $n_e(r)$  is an increasing function towards the center of the dipole magnetic field,  $N_e(r)$  is a decreasing function, hence interchange modes are stable. Note that the simple kinetic model predicts a flat distribution of  $N_e$  [9, 10]. The reason why  $N_e(r)$  decreases towards the center is explained by the thermodynamic equilibrium model to be described in section 3.

## 2.2. Pure electron plasma

RT-1 can also produce a pure electron plasma by injecting electrons from an electron gun placed at the periphery of the confinement region. A rotating electron cloud (confining typically  $10^{-8}$  C of electrons) sustains stably for more than 300 s [6, 7].

A single-species (non-neutral) plasma has a strong internal electric field. Confinement is possible only if the electric field ( $\mathbf{E}$ ) is balanced by an induction ( $\mathbf{v} \times \mathbf{B}$ ) generated by a vortical motion ( $\mathbf{v}$ ) in the magnetic field ( $\mathbf{B}$ ). The flow and the electromagnetic field achieve stable coupling by self-organizing a vortex (galaxies are close cousins, in which rotation creates a balance between gravitational and centrifugal forces). The non-triviality of *confinement* in the dipole magnetic field may be highlighted by comparing its formation process with that of conventional traps; the most popular method is to inject particles along a straight homogeneous magnetic



**Figure 4.** The radial profile of the space potentials inside an electron plasma measure by a high-impedance Langmuir probe (for the original data, see figure 9 of [7]). The solid line shows the space potential profile that corresponds to rigid-rotor  $\mathbf{E} \times \mathbf{B}$  rotation with the observed angular frequency  $\omega = 1.9 \times 10^5$  rad/s. The electron gun was located at  $r = 80$  cm and operated with acceleration voltage 500 V. The chain line is the potential profile calculated from the measured electron density.

field and then plug the entrance to the magnetic bottle by applying an electric field. The injected particles form an axisymmetric rigidly-rotating column [14, 15]. Magnetospheric non-neutral plasma is produced through an entirely different process. We simply emit electrons from an electron gun placed in a peripheral region of a static dipole magnetic field; we do not need to control the electric or magnetic field. Particles penetrate into the magnetosphere automatically and *self-organize* a stable vortical structure.

The self-organization process is driven by the Kelvin-Helmholtz (or, diocotron) instability, and continues until the plasma establishes a rigidly rotating vortex in which the instability quenches [6]. As shown in figure 4, such a structure is only possible if electrons diffuse inward, climbing up the potential hill—the internal electric potential is higher than the initial kinetic energy (acceleration voltage of the gun) of injected electrons.

### 3. Theoretical model of dipole confinement

In this section, we describe the self-organized confinement of the dipole configuration as a thermodynamic equilibrium in an effective phase space of magnetized particles which is “distorted” by the inhomogeneous magnetic field; the maximum entropy equipartition state has an inhomogeneous density when it is immersed in the laboratory flat space.

#### 3.1. Effective Hamiltonian of magnetized particles

Let us start by reviewing the effective Hamiltonian of a magnetized particle. The Hamiltonian of a charged particle is  $H = mv^2/2 + q\phi$ , where  $\mathbf{v} := (\mathbf{P} - q\mathbf{A})/m$  is the velocity,  $\mathbf{P}$  is the canonical momentum,  $(\phi, \mathbf{A})$  is the electromagnetic 4-potential,  $m$  ( $q$ ) is the particle mass (charge). Denoting by  $\mathbf{v}_{\parallel}$  and  $\mathbf{v}_{\perp}$  the parallel and perpendicular (with respect to the local magnetic field) components of the velocity, we may write

$$H = \frac{m}{2}v_{\perp}^2 + \frac{m}{2}v_{\parallel}^2 + q\phi. \quad (1)$$

The velocities are related to the mechanical momentum as  $\mathbf{p} := m\mathbf{v}$ ,  $\mathbf{p}_\parallel := m\mathbf{v}_\parallel$ , and  $\mathbf{p}_\perp := m\mathbf{v}_\perp$ . In a strong magnetic field,  $\mathbf{v}_\perp$  can be decomposed into a small-scale cyclotron motion  $\mathbf{v}_c$  and a macroscopic guiding-center drift motion  $\mathbf{v}_d$ . The periodic cyclotron motion  $\mathbf{v}_c$  can be *quantized* to write  $mv_c^2/2 = \mu\omega_c(\mathbf{x})$  in terms of the magnetic moment  $\mu$  and the cyclotron frequency  $\omega_c(\mathbf{x})$ ; the adiabatic invariant  $\mu$  and the gyration phase  $\vartheta_c := \omega_c t$  constitute an action-angle pair. For an axisymmetric dipole system with a poloidal (but no toroidal) magnetic field, we define a magnetic coordinate system  $(\psi, \zeta, \theta)$  such that  $\mathbf{B} = \nabla\psi \times \nabla\theta = B\nabla\zeta$  ( $\theta$  is the toroidal angle). The macroscopic part of the perpendicular kinetic energy is expressed as  $mv_d^2/2 = (P_\theta - q\psi)^2/(2mr^2)$ , where  $P_\theta$  is the angular momentum in the  $\theta$  direction and  $r$  is the radius from the geometric axis. In terms of the canonical-variable set  $\mathbf{z} = (\vartheta_c, \mu, \zeta, p_\parallel, \theta, P_\theta)$  the effective Hamiltonian (the guiding center Hamiltonian) becomes

$$H_c = \mu\omega_c + \frac{1}{2m} \frac{(P_\theta - q\psi)^2}{r^2} + \frac{1}{2m} p_\parallel^2 + q\phi. \quad (2)$$

The energy of the cyclotron motion has been quantized in term of the frequency  $\omega_c(\mathbf{x})$  and the action  $\mu$ ; the gyro-phase  $\vartheta_c$  has been coarse grained (integrated to yield  $2\pi$ ).

The standard Boltzmann distribution function is derived if we assume that  $d^3v d^3x$  is an invariant measure and the Hamiltonian  $H$  is the unique determinant of the ensemble. Maximizing the entropy  $S = -\int f \log f d^3v d^3x$  with constraining the total energy  $E = \int H f d^3v d^3x$  and the total particle number  $N = \int f d^3v d^3x$ , we obtain

$$f(\mathbf{x}, \mathbf{v}) = Z^{-1} e^{-\beta H}, \quad (3)$$

where  $Z$  is the normalization factor. The corresponding configuration-space density,

$$\rho(\mathbf{x}) = \int f d^3v \propto e^{-\beta q\phi}, \quad (4)$$

becomes constant for a charge neutral system ( $\phi = 0$ ).

Needless to say that the Boltzmann distribution or the corresponding configuration-space density, with an appropriate Jacobian multiplication, is independent of the choice of phase-space coordinates. Moreover, the density is invariant no matter whether we quantize the cyclotron motion or not. Let us confirm this fact by a direct calculation. For the Boltzmann distribution of the ‘‘guiding-center plasma’’

$$f(\mu, v_d, v_\parallel; \mathbf{x}) = Z^{-1} e^{-\beta H_c}, \quad (5)$$

the density is given by

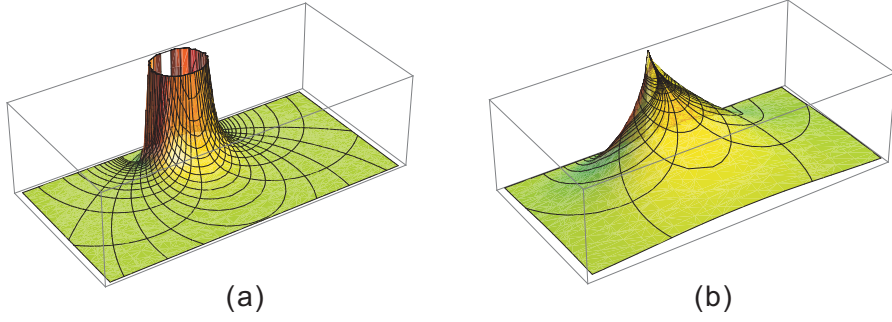
$$\rho(\mathbf{x}) = \int f d^3v = \int f \frac{2\pi\omega_c}{m} d\mu dv_d dv_\parallel \propto e^{-\beta q\phi}, \quad (6)$$

exactly reproducing (4).

### 3.2. Effective phase space and distorted metric

What makes the distribution function fundamentally different is the ‘‘constraints’’ on the phase space which limits the actual domain where the particles can occupy; the adiabatic invariants pose such constraints.

The constancy of the magnetic moment  $\mu$  imposes the strongest constraint. In addition, the action  $J_\parallel$  of the bounce motion (parallel to magnetic field lines) is also an



**Figure 5.** (a) Distorted coordinate-space metric (equivalent to the reciprocal flux-tube volume) in a dipole magnetic field. (b) Thermodynamic equilibrium on the constrained phase space; by immersing in the laboratory flat space, we obtain a heterogeneous density distribution.

adiabatic invariant that plays an essential law in the strongly inhomogeneous dipole magnetic field. To find explicit expressions for the parallel action-angle variables, we invoke the Hamiltonian  $H_c$  of (2) in which the gyration action-angle pair  $\mu$ - $\vartheta_c$  is “quantized” in a sense that  $\omega_c = \vartheta_c$  is given as a function of  $\mathbf{x}$  (configuration space coordinate). Neglecting the curvature effect and assuming charge neutrality ( $\phi = 0$ ), the equation of the parallel motion reads as

$$m \frac{d^2}{dt^2} \zeta = -\mu \nabla_{\parallel} \omega_c. \quad (7)$$

In the vicinity of  $\zeta = 0$ , where  $\omega_c$  has a minimum on each magnetic surface, we may expand  $\omega_c = \Omega_c(\psi) + \Omega_c''(\psi)\zeta^2/2$ , where  $\Omega_c(\psi)$  is the minimum of  $\omega_c$  and  $\Omega_c''(\psi) := d^2\omega_c/d\zeta^2|_{\psi}$ . In terms of the length  $L_{\parallel}(\psi) := [2\Omega_c(\psi)/\Omega_c''(\psi)]^{1/2}$ , which scales the variation of  $\omega_c$  along  $\zeta$ , (7) is integrated to identify the corresponding action-angle variables:  $\zeta = \ell_{\parallel} \sin \vartheta_{\parallel}$ ,  $\vartheta_{\parallel} = \omega_b t$  with the bounce frequency  $\omega_b = \sqrt{\Omega_c''(\psi)\mu/m} = v_{\perp}/L_{\parallel}(\psi)$ . The bounce amplitude  $\ell_{\parallel} = [2E_{\parallel}/(m\omega_b^2)]^{1/2}$  is evaluated in terms of the parallel energy  $E_{\parallel} := (mv_{\parallel}^2)/2|_{\zeta=0} = J_{\parallel}\omega_b$ . Assuming  $E_{\parallel} \approx E_{\perp} := \mu\Omega_c$ , we estimate  $\ell_{\parallel} \approx L_{\parallel}$ . The action  $J_{\parallel} := \oint mv_{\parallel}d\zeta/(2\pi)$  is related to  $E_{\parallel} = J_{\parallel}\omega_b$ , and  $dv_{\parallel} = (\omega_b/mv_{\parallel})dJ_{\parallel} = [\omega_b/(2mJ_{\parallel})]^{1/2}dJ_{\parallel}$ ; the latter, using the relation  $\omega_b/(mv_{\parallel}) = v_{\perp}/(L_{\parallel}mv_{\parallel}) \approx 1/(mL_{\parallel})$ , becomes  $dv_{\parallel} \approx (1/mL_{\parallel})dJ_{\parallel}$ .

The equilibrium distribution function is derived by maximizing entropy  $S$  over a micro-canonical ensemble of a given particle number  $N$ , an energy  $E$ , a total magnetic moment  $M_c = \int \mu f d^6z$  and a total bounce action  $M_b = \int J_{\parallel} f d^6z$ :

$$f_{\alpha,\gamma} = Z^{-1} e^{-(\beta H_c + \alpha \mu + \gamma J_{\parallel})}, \quad (8)$$

where  $\beta$ ,  $\alpha$ ,  $\gamma$  and  $\log Z - 1$  are, respectively the Lagrange multipliers on  $E$ ,  $M_c$ ,  $M_b$ , and  $N$ . Or, interpreting this  $f_{\alpha,\gamma}$  as a distribution function of a grand-canonical ensemble,  $\beta$  is the inverse temperature,  $\alpha/\beta$  and  $\gamma/\beta$  are the *chemical potentials* associated with the quasi-particles carrying microscopic actions  $\mu$  and  $J_{\parallel}$ .

The corresponding coordinate-space density is

$$\begin{aligned} \rho &= \int f_{\alpha,\gamma} \frac{2\pi\omega_c d\mu}{m} \frac{dJ_{\parallel}}{mL_{\parallel}(\psi)} dv_d \\ &\propto \frac{\omega_c(\mathbf{x})}{m^2} \int_0^{\infty} \frac{e^{-(\beta\omega_c + \alpha)\mu} d\mu}{\beta \sqrt{2\omega_c\mu/m} + \gamma L_{\parallel}(\psi)}. \end{aligned} \quad (9)$$

Notice that the Jacobian weight  $\propto \omega_c/L_{\parallel}(\psi)$ , representing the distortion of the metric on the constrained effective phase space, introduces a coordinate-space inhomogeneity; see figure 5. As shown in figure 3, the electron density is strongly peaked in the laboratory frame, but is hollow when multiplied by the flux-tube volume  $\oint d\ell/B \propto L_{\parallel}/\omega_c$  ( $\bar{x}$  denotes the orbit average of  $x$ ) —the decrease in the core (high  $\omega_c$ ) region, avoiding the divergence in the limit of the flux-tube volume  $\rightarrow 0$ , is due to the Boltzmann factor  $e^{-\beta\omega_c\mu}$  with a finite temperature  $\beta^{-1}$  (which was omitted in previous models).

#### 4. Summary

We have obtained experimental proof of the *self-organization* and *inward diffusion* in plasmas (both high-temperature, quasi-neutral and pure electron, non-neutral) which are spontaneously confined in a dipole magnetic field. The creation of a strongly localized density profile is explained by *distortion* of the metric on the effective (or, macroscopic) phase space; the maximum-entropy, equipartition distribution on the distorted phase space has inhomogeneous density when it is immersed in the laboratory flat space.

#### Acknowledgments

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