Dipole Confinement

theoretical understanding & experimental evidences

Zensho YOSHIDA

representing

The RT-1 Project

The University of Tokyo

○ I am going to present a concise summary of the RT-1 project of the University of Tokyo, which started at the suggestion of Akira Hasegawa about the dipole confinement.

○ There is, however, a bit longer history of our group; that is the study on the self-organization phenomena in nonlinear systems.

Before the RT-1 project, we had a Reversed-Field Pinch project that was the study of the so-called Taylor relaxed state of MHD plasma.

○ The connection of there two systems, which are seemingly very different, is in fact the main aim of my talk, and, indeed, it is also a clue to understand Akira's history of researches.

Self-organization

Akira's Philosophical Intuition

無為自然の核融合

fusion by spontaneous action

老子　(Lao Tzu)

References

[1] A. Hasegawa, C. G. Maclennan, Y. Kodama, *Nonlinear behavior and turbulence spectra of drift waves and Rossby waves*, Phys. Fluids 22 (1979), 2122.

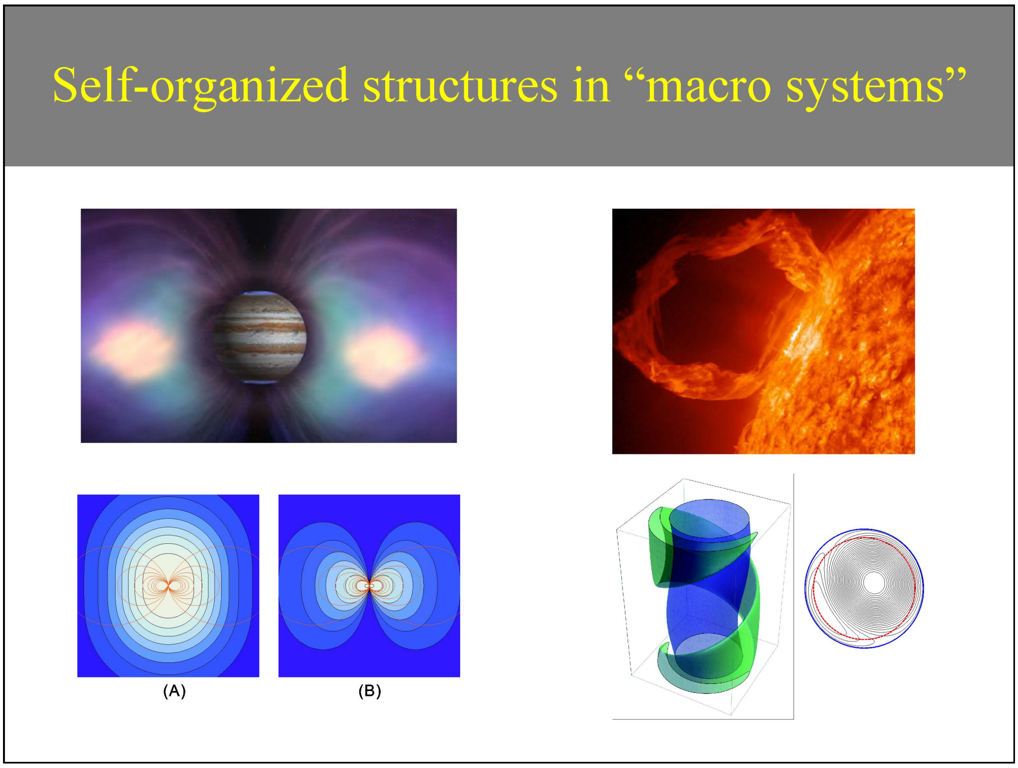
[2] A. Hasegawa, *Self-organization processes in continuous media*, Adv. Phys. 34 (1985), 1.

[3] A. Hasegawa, *Motion of a Charged Parand Plasma Equilibrium in a Dipole Magnetic Field – Can a Magnetic Field Trap a Charged Particle? Can a Magnetic Field Having Bad Curvature Trap a Plasma Stably?,* Phys. Scr. T116 (2005), 72.

○　Akira had a unique understanding of nature that is linked to antient Chinese thought.

○　This is a picture of Lao Tzu;

Akira often cited Lao Tzu's idea of naturalness to explain his ideas of self-organization of zonal flow, dipole confinement, as well as some other self-organization phenomena.



○ I will put his intuition into the perspective of Hamiltonian mechanics, and explain how these seemingly different phenomena, left is the dipole confinement and the right is the MHD relaxed state, they can be understood in the context of general physics theory.

Theoretical understanding

- *Invariants* are the key to the self-organization of non-trivial structures.
- *Hamiltonian mechanics* teaches "invariant = symmetry"
- *Casimir* is an intrinsic invariant, independent of any symmetry of a Hamiltonian.
- *Casimir = adiabatic invariant = symmetry in hidden micro variable.*
- Dipole confinement = confinement to a *leaf of Casimir*.

References

[1] ZY, *Self-organization by topological constraints: hierarchy of foliated phase space*, Adv. Phys. X **1** (2016), 2-19.

[2] ZY & S.M. Mahajan, *Self-organization in foliated phase space: Construction of a scale hierarchy by adiabatic invariants of magnetized particles*, PTEP **2014** (2014), 073J01.

○ The key to reveal the underlying mechanism of self-organization is the "invariants".

○ I will explain the notion of "invariants" in the perspective of Hamiltonian mechanics, the most established framework of dynamics theory. It teaches that an invariant is related to a symmetry.

○ However, the symmetry pertinent to my talk is somewhat non-standard one, which is called "Casimir invariant".
I proffer a theory of connecting a Casimir invariant with an "adiabatic invariant" that is generated by a symmetry of microscopic variable.

General Hamiltonian system

- Hamiltonian mechanics is dictated by a Poisson operator J and a Hamiltonian H.

$$\dot{u} = J\partial_u H(u)$$
$$\dot{G} = \{G, H\}$$

with Poisson bracket

$$\{G, H\} = \langle \partial_u G, J\partial_u H \rangle$$

$$\text{canonical} = \text{symplectic} \quad J = J_c = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

○ Let us first review the fundamentals of Hamiltonian mechanics.
○ In general, physics theory consists of two elements; one is the theory of space time, and the other is the theory of matter.
Hamiltonian mechanics dictates the space-time by the symplectic geometry that is structured by the Poisson algebra:
And formulate matter as a Hamiltonian.
○ The Poisson algebra is a Lie algebra endowed with a Poisson bracket ,that is written as this with an antisymmetric operator J, which we call the Poisson operator.
○ The textbook Poisson operator is the well-known symplectic matrix.

Non-canonical Hamiltonian mechanics

- Ker(J) = Coker(J) → "topological constraint"

$$0 = \langle v, J u_0 \rangle = \langle J^* v, u_0 \rangle = -\langle J v, u_0 \rangle \quad (u_0 \in \mathrm{Ker}(J),\ \forall v)$$

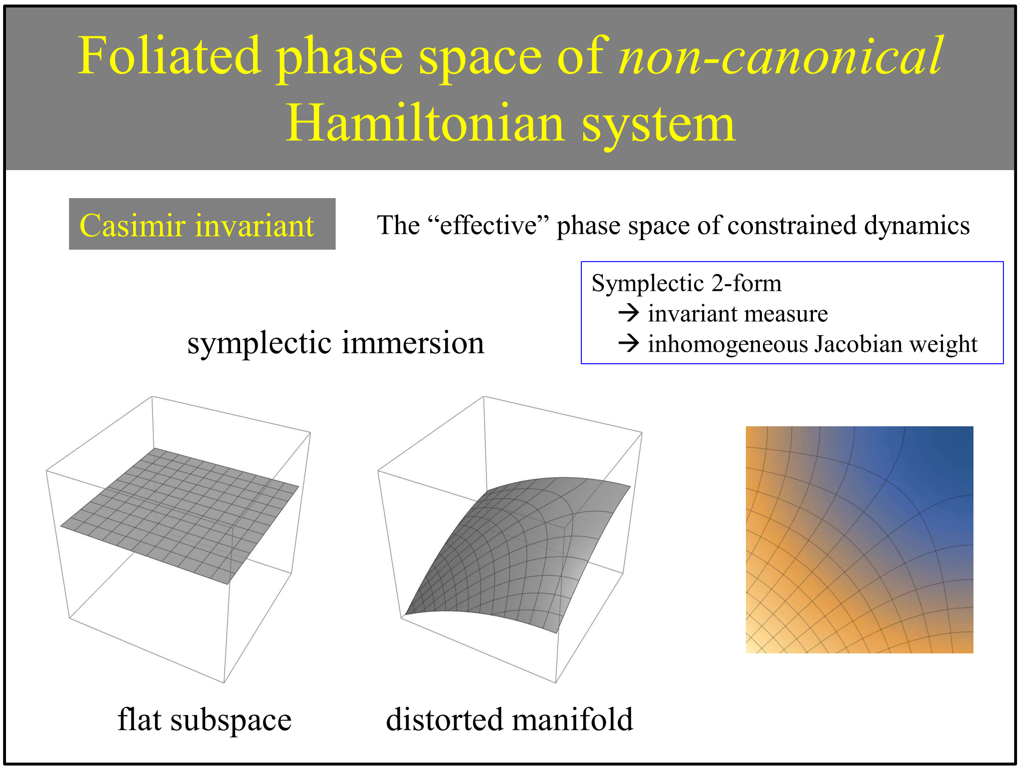
- Integrate Ker(J) → Casimir elements

$$\boxed{\partial_u C \in \mathrm{Ker}(J)} \Rightarrow \boxed{\{C, G\} = 0 \quad (\forall G)}$$

- Lie-Darboux theorem:

$$J = \begin{pmatrix} 0 & 0 \\ 0 & J_c \end{pmatrix} \qquad \text{(locally)}$$

○ A more general J may have a nontrivial kernel:
It is such a nullity of J, that yields the so-called Casimir invariant.
○ A Casimir invariant is a constant of motion generated by the degeneracy of J, so that cannot be changed by any Hamiltonian.
○ Notice that the usual invariant pertains to some symmetry of a Hamiltonian.
But a Casimir invariant is pertinent to the algebra of the space-time.
As I will explain, a Casimir invariant is typical for macroscopic systems, and is appropriate to interpret as an adiabatic invariant.
○ By Lie-Darboux theorem, we can, at least locally, every J into a standard form such as ///
The effective phase space is reduced by the rank of this nullity.



○ The effective phase space is a sub-manifold immersed in the naïve phase space.
○ The sub-manifold, which is called a leaf, must be a symplectic manifold.
○ An interesting situation occurs when the symplectic immersion yields a curved manifold, on which the metric, the invariant measure for Liouville's theorem, is distorted.
○ Then, the homogeneous distribution, maximizing the entropy, may have an inhomogeneous density when pull-baked to the original naïve phase space.
○ This is a general model of self-organization by topological constraint due to a defect of the underlying geometry.
○ What we observe as an inhomogeneous density in the original space is the Jacobian weight of the pull-back.

Theoretical understanding of the dipole confinement

- Inquiring into a kinetic model of charged particles, we show that

  Adiabatic invariant = Casmir invariant

  Macroscopic scale hierarchy = Casimir leaf

From here, we put forgoing general model of self-organization by examining the dipole plasma.

Scale hierarchy of magnetized particles

$$H = \frac{m}{2}\left(V_c^2 + V_\parallel^2 + V_d^2\right) + q\phi$$

$$H_c = \mu\omega_c + \frac{m}{2}\left(V_\parallel^2 + V_d^2\right) + q\phi$$
$$= \mu\omega_c + \frac{p_\parallel^2}{2m} + \frac{\left(P_\theta - q\psi\right)^2}{2mr^2} + q\phi$$

Decomposing the velocity into three components, the gyromotion, the bounce motion, and the toroidal drift motion, the Hamiltonian can be written in this form, after integrating the gyromotion as the action-angler frequency pair.

Coarse-graining

$$z = (\vartheta_c, p_c; \zeta, p_\parallel; \theta, P_\theta) \to (\cancel{\vartheta_c}, \mu; \zeta, p_\parallel; \theta, P_\theta)$$

Coarse-graining → non-canonicalization

$$J = \begin{pmatrix} J_c & 0 & 0 \\ 0 & J_c & 0 \\ 0 & 0 & J_c \end{pmatrix} \to J_{nc} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & J_c & 0 \\ 0 & 0 & J_c \end{pmatrix}$$

6 × 6 5 × 5

○ The coarse-graining of the gyro-angle means modifying the canonical Poisson matrix to a non-canonical one as this.

○ The Casimir invariant corresponding to the nullity is the magnetic moment.

Here, we see an explicit example in which a Casimir is an adiabatic invariant.

○ In general, we may assume a hidden conjugate variable associated with a Casimir.

It is a “hidden symmetry” in a macro-hierarchy, so the symmetry pertinent to a Casimir parallels the micro scale.

Grand-canonical distribution on a Casimir leaf

$$\delta(S-\alpha N-\beta E-\gamma M)=0$$

$$S=-\int f\log f\, d^6z$$

$$N=\int f\, d^6z$$

$$E=\int H_c f\, d^6z$$

$$M=\int \mu f\, d^6z$$

$$f=c\exp(-\beta H_c-\gamma\mu)$$

Chemical potential　　Quasi-particle number

○ The thermodynamic equilibrium on the leaf of the Casimir, the magnetic moment, is given by maximizing the entropy with an additional constraint on the Casimir.

○ Invoking the quantum-mechanical interpretation that action, the area surrounded by a periodic orbit in the phase space parallels particle number of quantum, we may interpret the Lagrange multiplier ¥gamma on the action ¥mu is a chemical potential, which measure the energy associated with a unit magnetic moment when it is introduced into the grand canonical ensemble.

Embedding into the lab-frame space

$$\rho(x) = \int f\, d^3v = \int f(2\pi\omega_c / m)\, d\mu\, dv_{\|}\, dv_\theta$$

$$= c\int \exp\left(-\beta H_c - \gamma\mu\right)\frac{2\pi\omega_c d\mu}{m} dv_{\|} dv_\theta$$

$$= \frac{\omega_c(x)}{\omega_c(x)+\gamma}$$

○ An interesting observation is obtained when we calculate the particle density in our naïve configuration space.

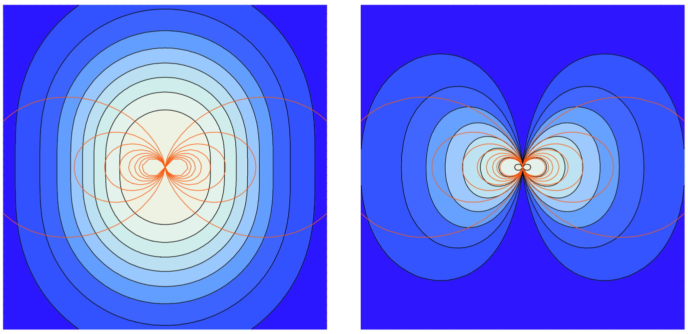
○ The density becomes high in the region where the magnetic field is high ---it is because the magnetic field is in the Jacobian of the coordinate transformation from the symplectic manifold to our Cartesian coordinates.

Jacobian weight in the lab-frame

Adiabatic invariants

→ *phase space* of quasi-particles

→ thermodynamic distribution on a Casimir leaf

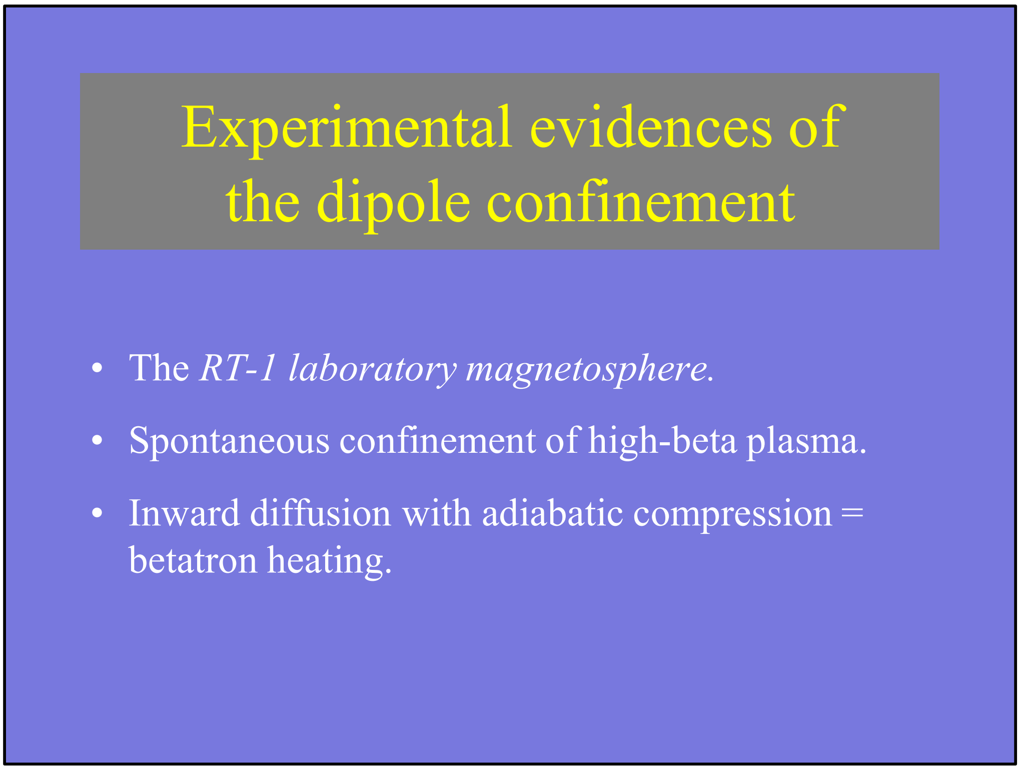


(A) (B)

○ The left figure shows the contour of the density in a dipole magnetic field.

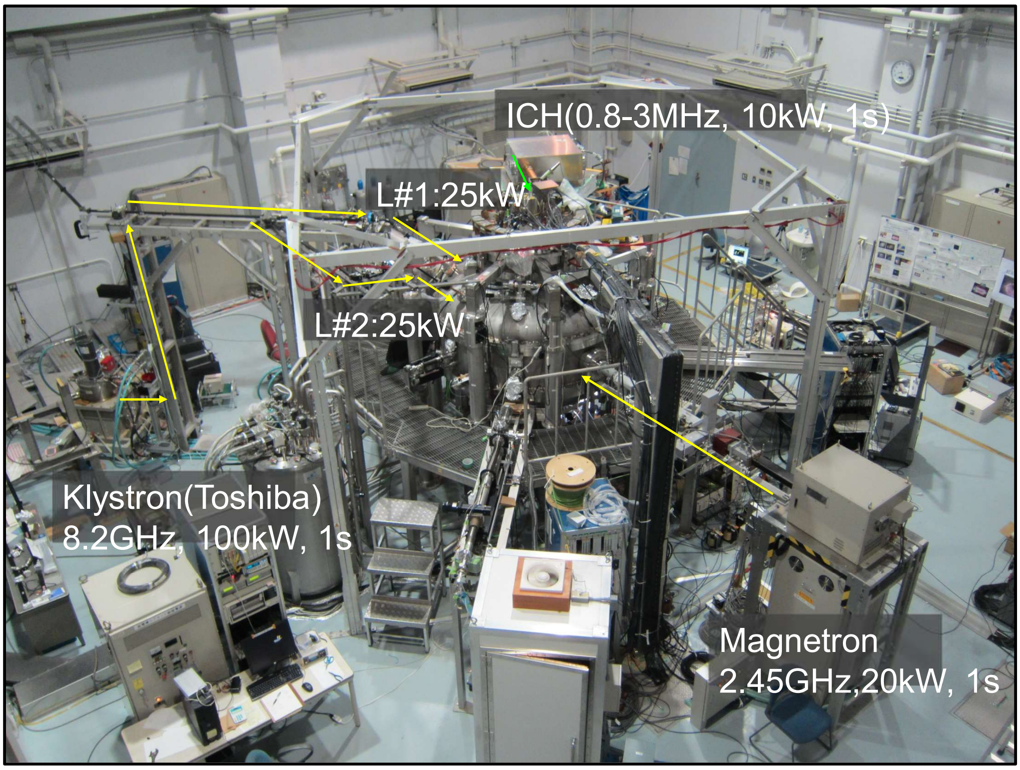
○ The right one is an equilibrium of further foliated leaf by taking into account the conservation of the parallel adiabatic invariant J_parallel.

○ This is indeed a good model of dipole plasma.



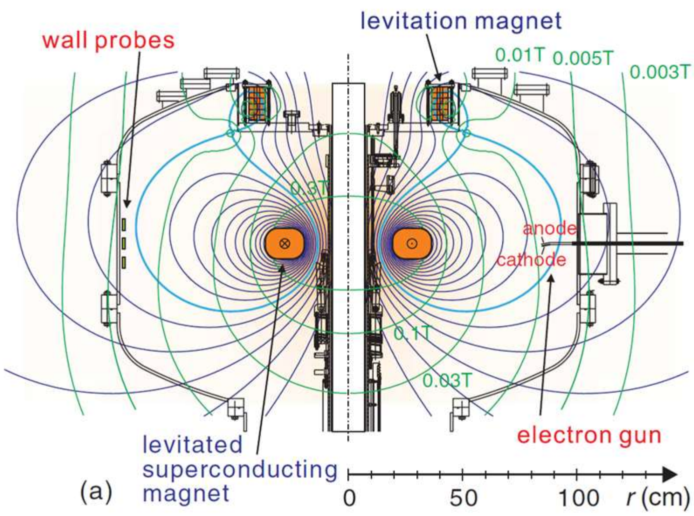
○ Here we describe the experimental demonstration of the dipole confinement.

○ We observe spontaneous inward, or up hill-diffusion, creating significantly peaked density profile,
Which is seemingly contradicting the entropy law, but is indeed a natural process on a symplectic leaf of constrained phase space.

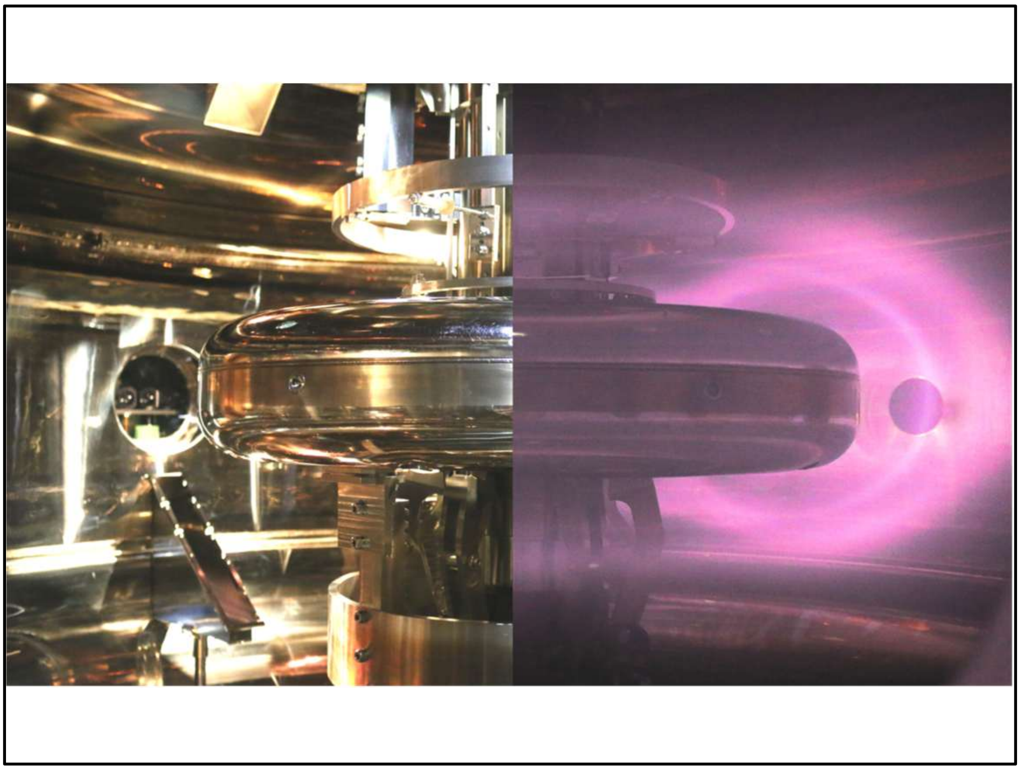


This is a photo of our RT-1 laboratory magnetosphere.

RT-1 experiment



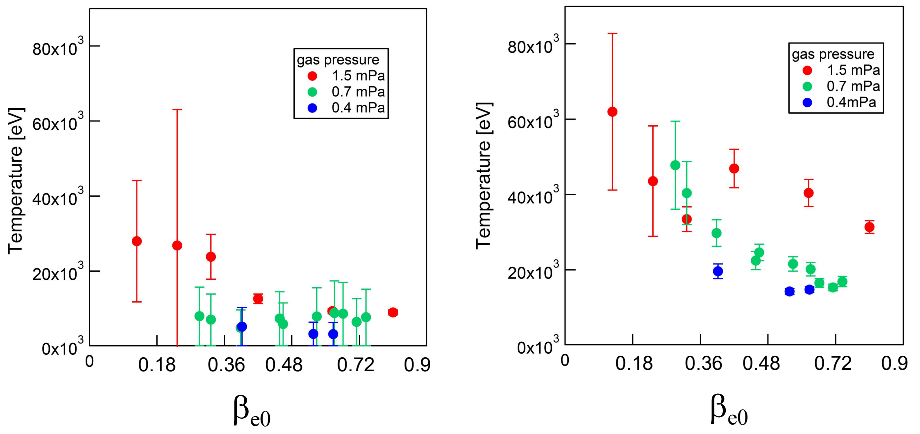
This is the vertical cross section of the device.
HTC superconducting magnet is levitated in a vacuum vessel of 1m radius.
Plasma is generated by ECH.



This collage shows the inside of the vessel with a ring magnet; the right-hand side is a picture of plasma.

High-beta confinement

Low energy component　　　　High energy component



M. Nishiura *et al., Improved beta (local beta >1) and density in electron cyclotron resonance heating on the RT-1 magnetosphere plasma,* Nucl. Fusion 55 (2015), 053019.

○　We have observed pretty good high-beta confinement.
Electrons have two temperature components; higher one typically 50keV, and lower one typically 5keV.
The central electron beta reaches about 1.
○　The electron energy confinement time is typically of order 0.1 sec.

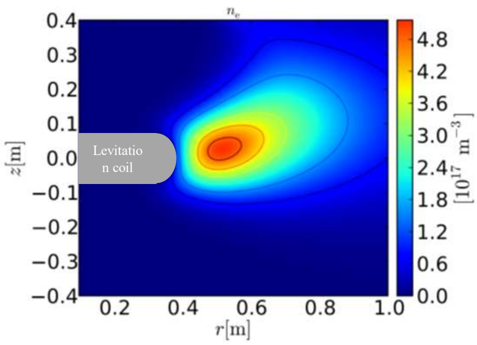
Inward diffusion

- In RT-1 magnetic field, resonance and cutoff layers are calculated for a typical high density discharge ($>10^{18}\mathrm{m}^{-3}$).
- Produced plasmas have a peaked density profile. This is explained by a theory related to the flux tube.

⇩

- The peaked density exceeds the cutoff density for 8.2 GHz.

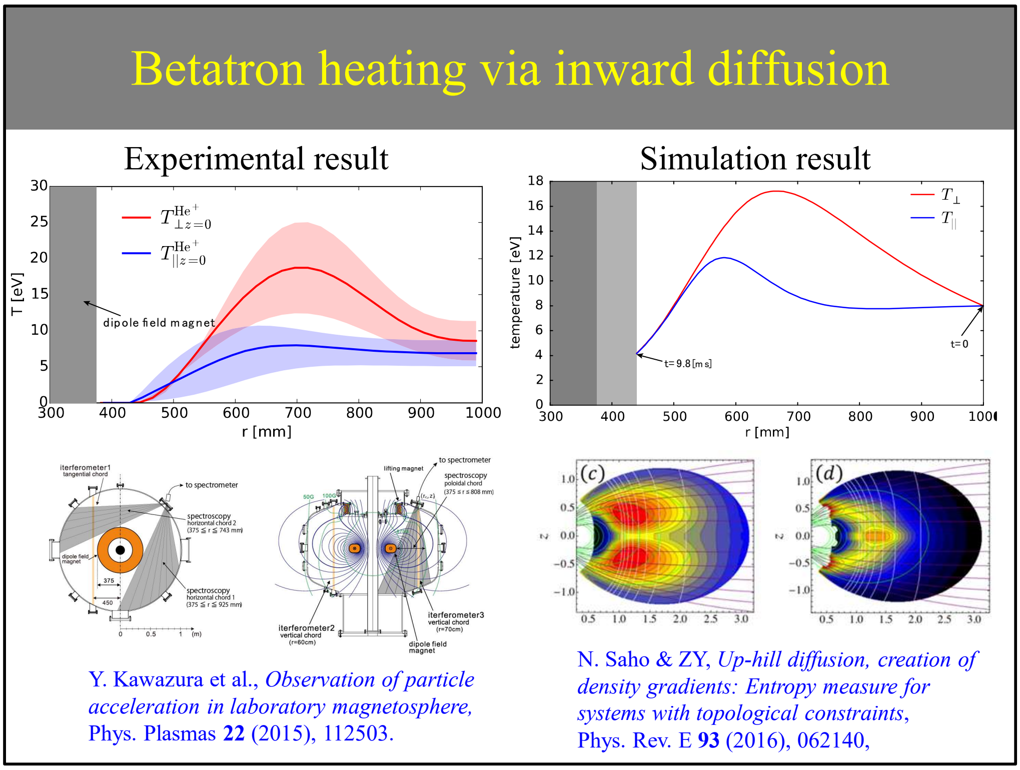
Inward diffusion mechanism explains the peaked density profile beyond the cutoff density.



Profile of the electron density reconstructed by three chord interferometer and edge probe.

H. Saito et al., *Measurement of a density profile of a hot-electron plasma in RT-1 with three-chord interferometry*, Phys. Plasmas **22** (2015), 024503.

The density peaks in the core region, inward from the ECH resonance.
This profile suggests the inward diffusion, relaxation toward “flat distribution” in the metric of the symplectic leaf.



○ There is another interesting observation that is connected to the inward diffusion.

○ The left figure shows the anisotropy of the ion temperature.
As shown, the perpendicular temperature is significantly higher than the parallel temperature.
Since we do not apply any direct heating for ions, there temperature is determined by heating by ambient electrons, which cannot explain such anisotropy, and some other collective phenomena.

○ The right figure shows a result of simulation that considers both betartron and Fermi acceleration mechanisms associated with the inner diffusion.
We observe that the inward diffusion toward stronger magnetic field region brings about betatron acceleration, due to the conservation of the magnetic moment, and enhances the perpendicular temperature.

Unification

dipole confinement, zonal flow & Taylor relaxed state

- Hamiltonian mechanics unifies the ideas of dipole confinement, zonal flow, and Taylor relaxation.
- The key is the “constraint” in the phase space of Hamiltonian mechanics.

○ Now we change the subject, and discuss another example of self-organization, i.e., the famous Taylor relaxed state of MHD.
○ The general framework of Hamiltonian mechanics that we have discussed at the beginning of this talk, we see that the Taylor relaxed state is an example of thermal equilibrium on the leaf of helicity in the phase space of MHD.

Hamiltonian form of MHD

- Hamiltonian

$$H(u)=\frac{1}{2}\int_\Omega\left(V^2+B^2\right)dx=\frac{1}{2}\|u\|^2$$

- Poisson operator

$$J(u)=\begin{pmatrix} -P_\sigma(\nabla\times \boldsymbol{V})\times\circ & P_\sigma(\nabla\times\circ)\times \boldsymbol{B} \\ \nabla\times(\circ\times \boldsymbol{B}) & 0 \end{pmatrix}$$

- *Casimir invariants:*

$$C_1(u)=\frac{1}{2}\langle \boldsymbol{A},\boldsymbol{B}\rangle,\quad C_2(u)=\langle \boldsymbol{V},\boldsymbol{B}\rangle$$

P. J. Morrison, Rev. Mod. Phys **70**, 467 (1998).

○ MHD is an infinite-dimensional Hamiltonian system with a somewhat frightening Poisson operator.
○ The well-known magnetic helicity is a Casimir of this Poisson algebra.

Beltrami equilibria on helicity leaves

- Beltrami equilibrium:

$$\partial_u H_\mu(u) = 0 \quad (H_\mu = H - \mu C_1).$$

$$\rightarrow \quad \nabla \times \boldsymbol{B} = \mu \boldsymbol{B}, \quad \boldsymbol{B} \in L^2_\sigma(\Omega).$$

- Two classes of Beltrami eigenvalues:

(1) Self-adjoint curl operator S

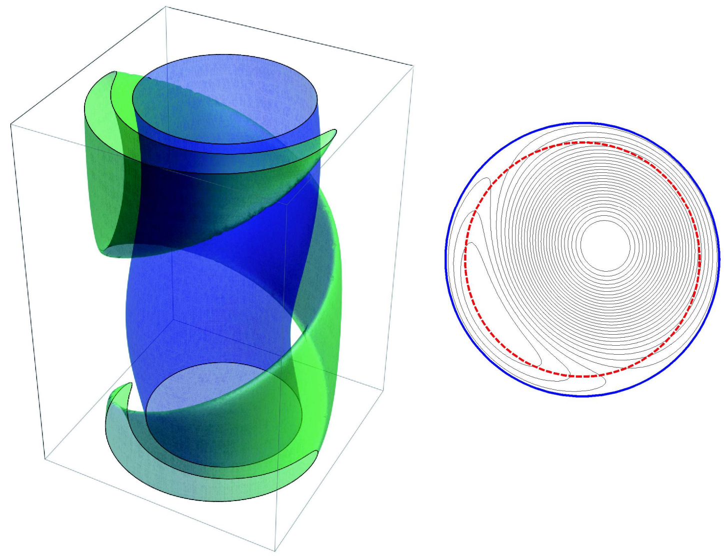
→ discrete real eigenvalues $\mu \in \{\lambda_1, \lambda_2, \cdots\}$

(2) Non-self-adjoint extension T
(if Ω is multiply connected) → $\forall \mu \in \mathbf{C}$

ZY & Y. Giga, Math. Z. **204** (1990) 235.

○ The equilibrium minimizes the energy with the constraint on the helicity.
○ The corresponding Euler-Lagrange equation reads as the eigenvalue problem of the curl operator; the eigenfunctions are called Beltrami fields, which consist the cohomology of the base space.

Taylor relaxed state = Beltrami equilibrium



This picture shows a typical "twisted" structure of the eigenfunction.

summary

- Variety of "self-organization phenomena" — *dipole confinement, Taylor relaxed state, zonal flow* — can be put into the perspective of "foliated phase space" of Hamiltonian mechanics.
- Macroscopic hierarchy = Casimir leaves = separation of "adiabatic invariants"

Let me summarize my talk.

○ Variety of self-organization, including the dipole confinement, Taylor relaxed state, as well as zonal flow, these are the thermodynamic equilibria on leaves of constrained Hamiltonian systems.

○ The Casimir invariants can be interpreted as adiabatic invariants, which defines "macro" hierarchy as foliated phase space.