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# Chaos of energetic positron orbit in a dipole magnetic field configuration

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### Conservation and violation of adiabatic invariants for <sup>2</sup> particles in dipole magnetic field, the case of high-energy e+

- Particle orbit in a dipole field basically consists of three periodic motions, making three adiabatic invariants as actions
- Adiabatic invariants are not always const.
   ex) slow fluctuations break \u03c8 conservation, while conserving \u03c4 and J, causing radial transport and acceleration of particles
- Collective behavior of particles under such constraint makes macroscopic self-organization of plasmas

2015 Yoshida Mahajan, Prog. Theor. Exp. Phys. 2019 Nishiura+ NF, 2015 Kawazura+ PoP, 2017 Sato+ PRE

 Breakdown of adiabaticity and orbit chaos appear even without fluctuations or asymmetry of the system:
 High-energy positrons in dipole field





### Non-linear term in H of e+, due to coupling between motion modes, violates the adiabaticity, making system non-integrable

When the three adiabatic invariants are conserved, the Hamiltonian is

 $H = \mu \omega_c + J \omega_b + \Psi \omega_d$ 

 $\omega_c, \omega_b, \omega_d$ : gyro, bounce, drift frequencies

 According to Arnold-Liouville theorem, this system is integrable, particles are trapped on periodic orbits in a dipole field

Number of first integral (independent conserved quantity) > degree of freedom

• Non-linear term violates the adiabaticity of  $\mu$ , *J*, and  $\Psi$ , through the effects of fluctuations, asymmetry, orbit mode coupling, etc.

$$H = \mu \omega_c + J \omega_b + \Psi \omega_d + N_c$$
1990 Murakami, Sato, Hasegawa, PoF

• For high energy positrons, gyro and bounce motions easily couple, which means there are only two constants of motion, H and  $P_{\theta} \sim \Psi$  (axisymmetry)

We numerically and experimentally investigate the chaotic behavior of positrons from Na-22 isotope in a dipole field

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## Chaotic motion of high-energy positrons is also 4/13 applicable to scientific application: electron-positron plasmas

APEX collaboration plans to create and study electron-positron plasmas



- Available positron beam is not very strong; efficient injection are essentially important
- Use of chaotic long orbit of high-energy positrons, together with intense Na-22 source can be alternative method

#### Chaotic long orbit of positrons is attractive

2002 Nakashima+ PRE



Reactor (FRM-II) based e+ source 2012 Hugenschmidt+, New J. Phys.

### Orbit and adiabatic invariant properties of positrons are investigated in the dipole configuration of RT1

 Positrons are injected from a Na-22 source located at the edge of the confinement region of RT1
 2016 Saitoh+, PRE



RT1 (Ring Trap 1), superconducting levitated dipole of Uni. Tokyo, Japan

 In a dipole field generated by a ring current of 250kA (R=0.25m), orbit is
 calculated by numerically integrating

$$\frac{d}{dt}(\gamma m\mathbf{v}) = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Relativistic adiabatic invariants:

$$\mu = \int \gamma v_{\perp} dl = \int \frac{\gamma^2 v_{\perp}^2}{B} dt$$
$$J = \int \gamma v_{\parallel} ds$$
$$\Psi = \int \mathbf{B} \cdot d\mathbf{S}$$

2006 Yoshida+ PFR, 2010 Yoshida+ PRL

## Regular (low energy) and non-regular (high energy) orbits 6/13 according to kinetic energy of e+



Top view of RT1, including typical orbits of positrons injected from Na-22 source

 Particle motion consists of gyro + bounce + toroidal drift

 Toroidal motion is realized by the curvature and grad-B drift

- Orbit properties are different according to particle energy
- 1keV: periodic, return to Na-22 source after one circulation
- 100keV: non-periodic motion



#### Mode coupling between gyro and bounce motions breaks The conservation of $\mu$ and *J*, making chaotic behavior



- µ and J are conserved
   for 1 keV positron
- motion is periodic
- gyro and bounce motions are separated
- μ and J are not conserved
   for 100 keV positron
- motion is non-periodic
- gyro and bounce motions are overlapped

 $\Psi$  is always conserved due to the asymmetry

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#### Conservation map for $\mu$ and J shows dependence on both kinetic energy and pitch angle of particle



0

2

4

6

time (µs)

8

10

particle energy and pitch angle

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 $10^2 \ 10^3$ 

frequency (MHz)

 $10^{0}$  $10^{1}$ 

## Breakdown of the first and second invariants results in the <sup>9/13</sup> chaotic motion of high-energy positrons



Poincaré plot of orbit in the phase space for different pitch angles for positrons of kinetic energy 1, 20, and 100 keV

- Considerable ratio of positrons from Na-22 source exhibits chaos in the geometry of the RT1 levitated dipole
- Such particles may have long orbit (i.e., long trapping time) in RT1 before annihilation by recombination at the source

## Effects of chaos: Positrons with non-integrable chaotic 10/13 trajectories have considerably long orbit before annihilation



Flight length of single particle:

- e+ in periodic motions return to source just after single gyro/bounce motion
- positrons with chaotic orbits make multiple toroidal circulation with longer flight time



## By minimizing the source size, thin torus e+ cloud is steadily generated, to be transported into strong field region



Flight time and length reflecting energy distribution:

• Na-22 source size ( $r_{source}$ ) limits the flight time

 $\frac{dN_1}{dt} = \int \Gamma dS - \frac{N_1}{\tau}$  particle balance inside the hollow cloud

- with a source of r<sub>source</sub> = 0.5cm (-> averaged flight time: 50 μs) and100 mCi (3.7 GBq), a hollow could of ~1×10<sup>4</sup> positrons are steadily generated
- if 1% of e+ from the source are transported inward by rotating wall, and assuming 1000 s trapping time,  $N_1 \sim 10^{10}$  is expected in strong field region

### Experiments at RT1 show fairly good agreement with numerical analysis, indicating the validity of chaos effects





• Positrons injected from a 1 MBq (27  $\mu$ Ci) Na-22 source at the edge confinement region

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- Target probe at the opposite side detect e+ by coincidence measurements of 511 keV γs
- Orbit calculations including the chaotic effects well reproduced the experimental results

#### Summary

- Non-linear behavior of high-energy positrons in a dipole field was investigated both numerically and experimentally.
- In a typical levitated dipole field experiment, such as RT-1, it was shown that considerable ratio of high-energy positrons from radioactive sources have non-integrable chaotic and long orbits.
- These particles make multiple toroidal precessions forming a hollow toroidal cloud, which may be applied for the formation of dense positron cloud in a strong magnetic field region.
- Experiments with a small Na-22 source in RT-1 demonstrated the existence of long-lived positrons in the dipole field, showing a good agreement with numerical orbit analysis.
- Such chaos effects would do exist and play some role for high-energy protons and electrons in magnetospheres.